Efficient Algorithms for Matching

Dan Huttenlocher and Phil Torr
ICCV 2003

Dynamic Programming For Detection

Fast Detection

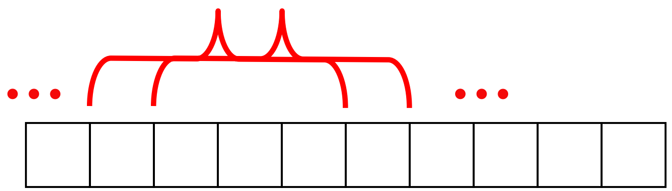
- For example finding faces at video rates

Dynamic Programming (DP)

- General algorithmic technique
  - Not specific algorithm
  - Analogous to “divide and conquer” – bottom up
- Methods that cache solutions to sub-problems rather than re-computing them
  - E.g., Fibonacci, substring matching
- Applies to problems that can be decomposed into sequence of stages
  - Each stage expressed in terms of results of fixed number of previous stages

Simple DP Example: Box Sum

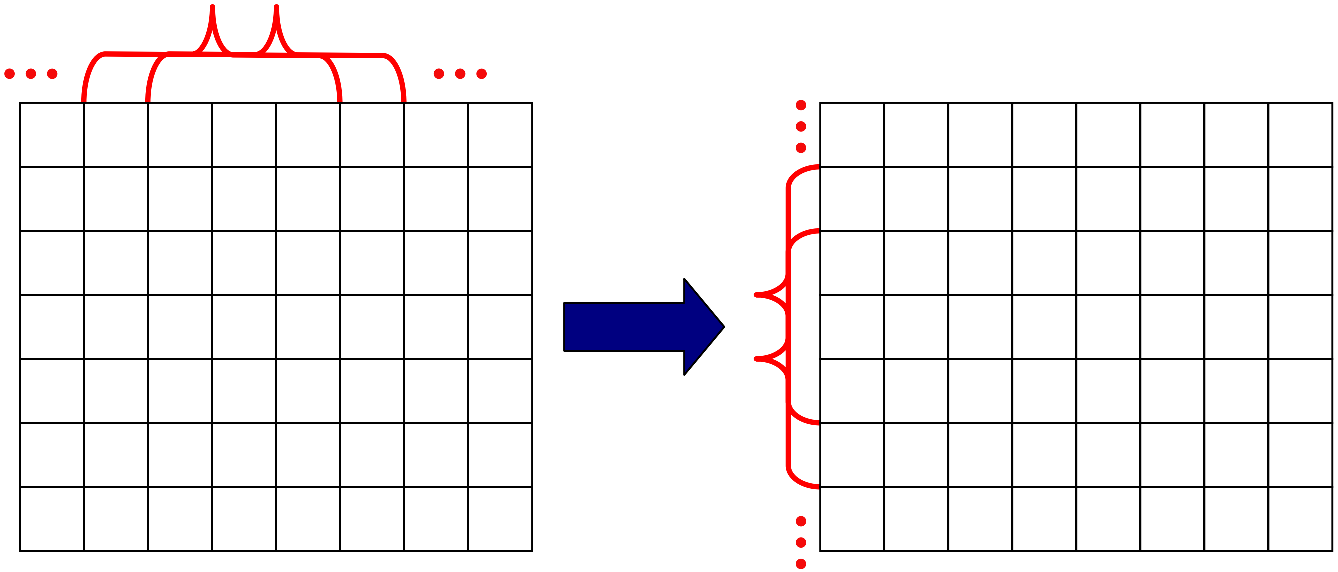
- Sum n-vector over sliding k-window
  - $W_k[x] = f[x] + \ldots + f[x+k]$
  - Note: often k odd, sum between $x \pm (k-1)/2$



- Explicit summation $O(k*n)$ additions
- Recurrence yields $O(n+k)$ time method
  - $W_k[x] = W_k[x-1] + f[x+k] - f[x-1]$
  - Each element of sum differs from previous by just two values

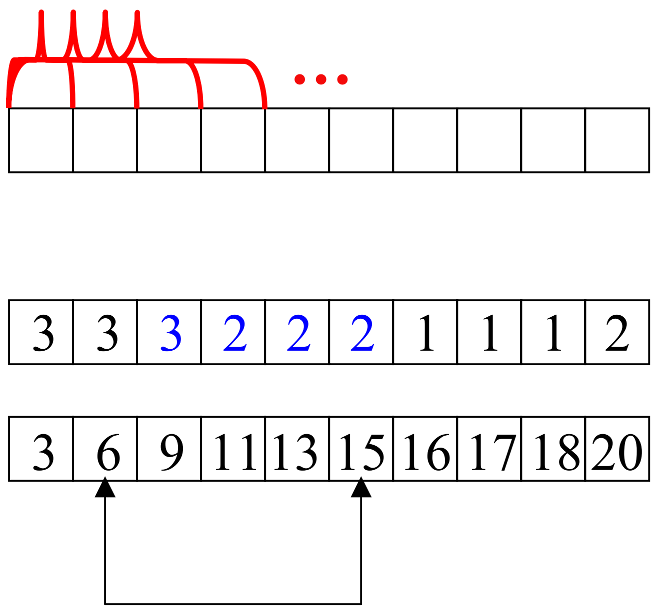
Box Sums in d Dimensions

- One pass along each dimension
  - Sum intermediate result from previous pass
  - 2D case: horizontal then vertical (or vice versa)
    - m by n image, O(mn+wh) time vs. O(mnwh)
    - E.g., 10 by 10 summation window, 100x faster



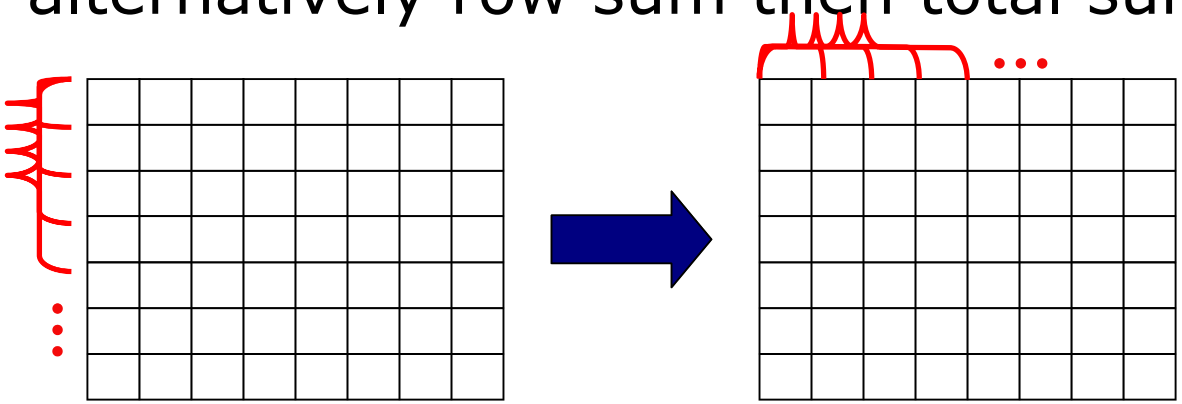
1d Integral Images

- Fast summations over different sized regions (non spatially uniform)
- Cumulative sum
  - $S[x] = f[0] + \ldots + f[x]$
- DP recurrence O(n) time
  - $S[x] = S[x-1] + f[x]$
- Sum over window of f[x] independent of size k
  - $W_k[x] = S[x+k-1]-S[k-1]$



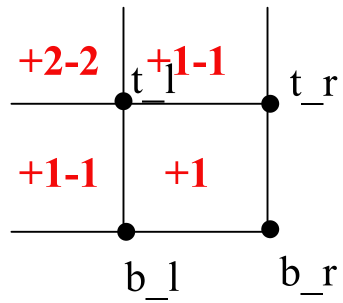
n-d Integral Images

- Analogous for higher dimensions, 2D:
  - S[x,y] = f[0,0] + … + f[0,y] + … f[x,0] + … + f[x,y]
- Separate recurrence per dimension
  - C[x,y] = C[x,y-1] + f[x,y]   (column sum)
  - S[x,y] = S[x-1,y] + C[x,y]   (total sum)
  - Or alternatively row sum then total sum



Fast Region Sums With II

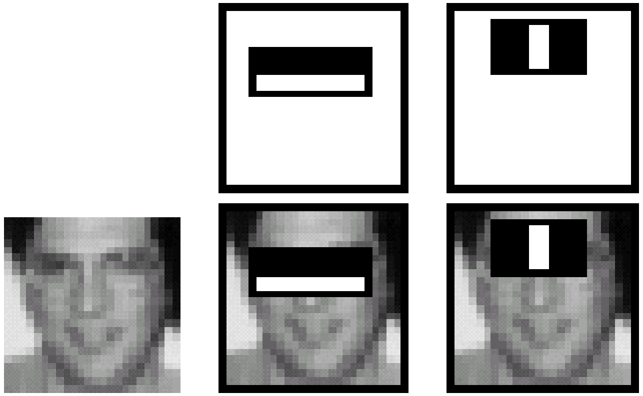
- Sum over a rectangle, constant time
  - S[b_r] + S[t_l-(1,1)] – S[b_l-(1,0)] – S[t_r-(0,1)]



- Sum over arbitrary region, linear time
  - Running time proportional to length of boundary not area

Fast Detection With II

- Features formed from combinations of sums over rectangles
  - For example positive and negative regions
  - Running time independent of rectangle size
- Viola and Jones use for face detection at approximately video rates



Fast Detection With II

- Also useful for arbitrary shaped regions
  - Decompose into rectangles
    - With no holes in worst case this is number of scan lines (not too bad with holes either)
    - Proportional to boundary length rather than area
  - Construct chain-code representation of boundary and sum values
    - Positive for downward links and negative for upward (reverse for holes)
  - Note relation to work of Jermyn and Ishikawa on boundary integrals

Distance Transforms

Distance Transforms

- Map of distance to nearest features
  - Computed from map of feature locations
    - E.g., edge detector output
- Powerful and widely applicable
  - Can think of as "smoothing in feature space"
  - Related to morphological dilation operation
  - Often preferable to explicitly searching for correspondences of features
- Efficiently computable using DP
  - Time linear in number of pixels, fast in practice

Distance Transform Definition

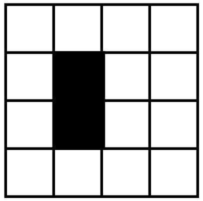
- Set of points, P, some distance $\|\bullet\|$

$$D_P(x) = \min_{y \in P} \|x - y\|$$

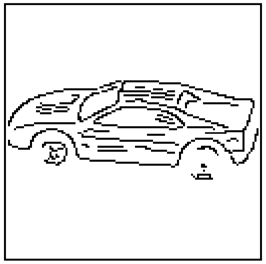
  - For each location x distance to nearest y in P
  - Think of as cones rooted at each point of P
- Commonly computed on a grid Γ using

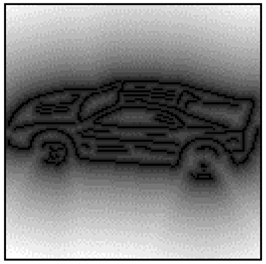
$$D_P(x) = \min_{y \in \Gamma} (\|x - y\| + 1_P(y))$$

  - Where $1_P(y) = 0$ when $y \in P$, ∞ otherwise



2	1	2	3
1	0	1	2
1	0	1	2
2	1	2	3





DP for L_1 Distance Transform

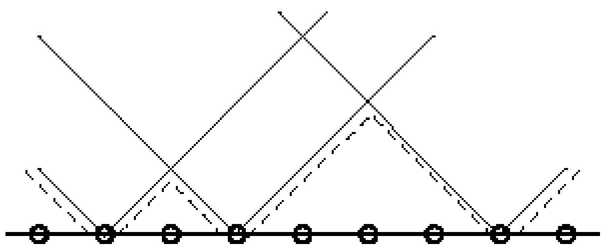
- 1D case
  - Two passes:
    - Find closest point on left
    - Find closest on right if closer than one on left
  - Incremental:
    - Moving left-to-right, closest point on left either previous closest point or current point
    - Analogous moving right-to-left for closest point on right
  - Can keep track of closest point as well as distance to it
    - Will illustrate distance; point follows easily

L_1 Distance Transform Algorithm

- Two pass O(n) algorithm for 1D L_1 norm (for simplicity just distance)
  1. Initialize: For all j
     $D[j] \leftarrow 1_{\mathbf{P}}[j]$
  2. Forward: For j from 1 up to n-1
     $D[j] \leftarrow \min(D[j], D[j-1]+1)$
  3. Backward: For j from n-2 down to 0
     $D[j] \leftarrow \min(D[j], D[j+1]+1)$

1	0

0	1



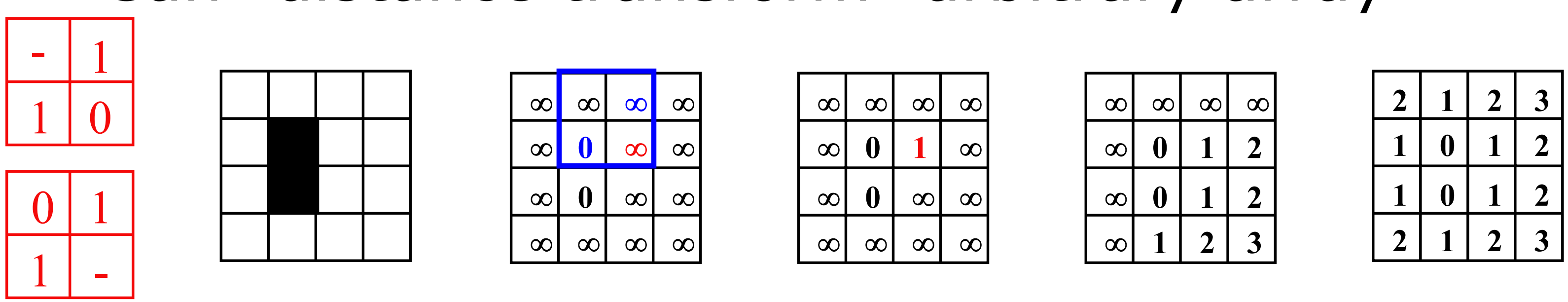
∞	0	∞	0	∞	∞	∞	0	∞

∞	0	1	0	1	2	3	0	1

1	0	1	0	1	2	1	0	1

L_1 Distance Transform

- 2D case analogous to 1D
  - Initialization
  - Forward and backward pass
    - Fwd pass finds closest above and to left
    - Bwd pass finds closest below and to right
- Note nothing depends on 0,∞ form of initialization
  - Can "distance transform" arbitrary array



L_2 Distance Transform

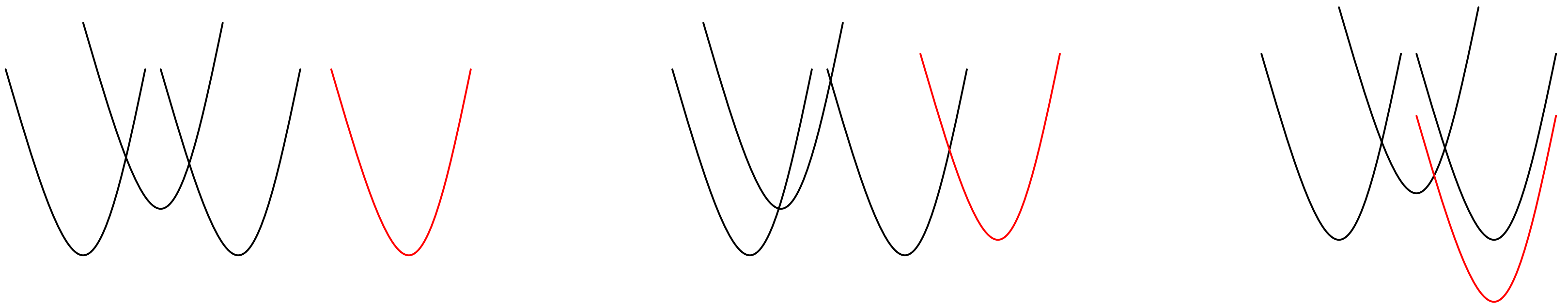
- Approximations using fixed size masks
  - Analogous to L_1 case
  - Simple to understand but not best methods
- Exact linear time method for L_2^2
  - Can compute sqrt (but usually not needed)
  - Fast in practice, easy to implement
  - Harder to understand than L_1 algorithm
  - Uses important general algorithmic technique of amortized analysis
- 1D case – lower envelope of quadratics

1D L_2^2 Distance Transform

- Single left-to-right pass
  - Adding k-th quadratic to lower envelope (LE) of first k-1 quadratics
  - Quadratics differ only in location of their base
- Concerned about intersection of k-th quadratic and LE of first k-1
  - Consider only rightmost quadratic visible in LE
  - Keep track of locations of bases of *visible quadratics* (VQ), ordered left-to-right
  - Keep track of *visible intersections* of adjacent quadratics (VI), ordered left-to-right

Adding k-th Quadratic to LE

- Case 1: intersection of k and rightmost VQ (RVQ) outside range, k not visible on LE
- Case 2: intersection of k and RVQ to right of rightmost VI (RVI), k added to right
- Case 3: intersection of k and RVQ to left of RVI, k covers at least RVQ, remove RVQ and try adding again



Running Time of 1D Algorithm

- Traditional analysis would consider time for each case, multiplied by n iterations
  - Cases 1 and 2 O(1), but case 3 ??
- <u>Amortized analysis</u>: charge work done by algorithm to "events" that can be bounded
  - Three event types
    - K-th quadratic initially excluded
    - K-th quadratic added
    - K-th quadratic removed
  - Each event happens at most once per quadratic (note once removed, never again)
  - Algorithm does constant work per event

2D Algorithm

- Horizontal pass of 1D algorithm
  - Computes minimum x^2 distance
- Vertical pass of 1D algorithm on result of horizontal pass
  - Computes minimum x^2+y^2 distance
  - Note algorithm applies to any input (quadratics can be at any location)
- Actual code straightforward and fast
  - Each pass maintains arrays of indexes of visible parabolas and the intersections
  - Fills in distance values at each pixel after determining which parabolas visible

Horizontal Pass of 2D L_2^2 DT

```
for (y = 0; y < height; y++) {
   k = 0;  /* Number of boundaries between parabolas */
   z[0] = 0;   /* Indexes of locations of boundaries */
   z[1] = width;  /* No current boundaries (first at end of array) */
   v[0] = 0;     /* Indexes of locations of visible parabola bases */
   for (x = 1; x < width; x++) {
     do {
    /* intersection of this parabola with rightmost visible parabola */
    s = ((imRef(im, x, y) + x*x) - (imRef(im, v[k], y) + v[k]*v[k])) /
      (2 * (x - v[k]));
     sp = ceil(s);
     /* case one: intersection off end, this parabola not visible */
     if (sp >= width)
       break;
     /* case two: intersection is rightmost, add it to end*/
     if (sp > z[k]) {
       z[k+1] = sp; z[k+2] = width; v[k+1] = x; k++;
       break;  }
     /* case three: intersection is not rightmost, hides rightmost
        parabola and perhaps others, remove rightmost and try again */
     if (k == 0) {
       v[0] = x; break;
     } else {
       z[k] = width; k--; }
     } while (1);
   }
```

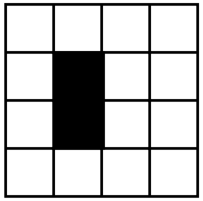
DT Values From Intersections

```
/* get value of input image at each parabola base */
for (x = 0; x <= k; x++) {
  vref[x] = imRef(im, v[x], y);
}
k = 0;
/* iterate over pixels, calculating value for closest parabola */
for (x = 0; x < width; x++) {
  if (x == z[k+1])
 k++;
  imRef(im, x, y) = vref[k] + (v[k]-x)*(v[k]-x);
}
```

- No reason to approximate L_2 distance!
- Code available at www.cs.cornell.edu/~dph/matchalgs/

DT and Morphological Dilation

- Dilation operation replaces each point of P with some fixed point set Q
  - $P \oplus Q = \cup_{\mathbf{p}} \cup_{\mathbf{q}}\, p+q$
- Dilation by a "disc" C^d of radius d replaces each point with a disc
  - A point is in the dilation of P by $C^{\mathbf{d}}$ exactly when the distance transform value is no more than d (for appropriate disc and distance fcn.)
  - $x \in P \oplus C^{\mathbf{d}} \Leftrightarrow D_{\mathbf{P}}(x) \leq d$



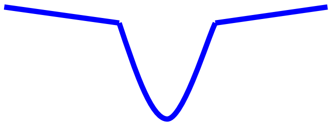
2	1	2	3
1	0	1	2
1	0	1	2
2	1	2	3

0	1	0	0
1	1	1	0
1	1	1	0
0	1	0	0

1	1	1	0
1	1	1	1
1	1	1	1
1	1	1	0

Generalizations of DT

- Combination distance functions
  - Robust "truncated quadratic" distance
    - Quadratic for small distances, linear for larger
    - Simply minimum of (weighted) quadratic and linear distance transforms



- DT of arbitrary functions: $\min_{\mathbf{y}} \| x-y \| + f(y)$
  - Exact same algorithms apply
  - Combination of cost function f(y) at each location and distance function
    - Useful for certain energy minimization problems

Distance Transforms in Matching

Distance Transforms in Matching

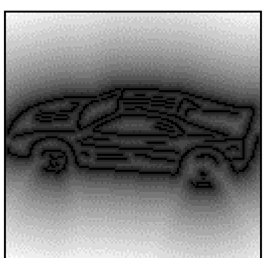
- Chamfer measure – asymmetric
  - Sum of distance transform values
    - "Probe" DT at locations specified by model and sum resulting values
- Hausdorff distance (and generalizations)
  - Max-min distance which can be computed efficiently using distance transform
  - Generalization to quantile of distance transform values more useful in practice
- Iterated closest point (ICP) like methods
  - Traditionally search for matches, DT faster

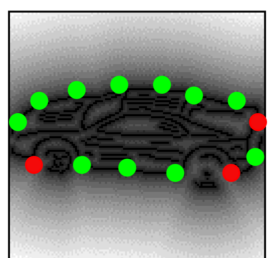
Hausdorff Distance

- Classical definition
  - Directed distance (not symmetric)
    - $h(A,B) = \max_{a \in A} \min_{b \in B} \| a-b \|$
  - Distance (symmetry)
    - $H(A,B) = \max(h(A,B), h(B,A))$
- Minimization term is simply a distance transform of B
  - $h(A,B) = \max_{a \in A} D_B(a)$
  - Maximize over selected values of DT
- Classical distance not robust, single "bad match" dominates value

Hausdorff Matching

- Partial (or fractional) Hausdorff distance to address robustness to outliers
  - Rank rather than maximum
    - $h_{\mathbf{k}}(A,B) = \text{kth}_{\mathbf{a \in A}} \min_{\mathbf{b \in B}} \| a\text{-}b \| = \text{kth}_{\mathbf{a \in A}} D_{\mathbf{B}}(a)$
  - K-th largest value of $D_{\mathbf{B}}$ at locations given by A
  - Often specify as fraction f rather than rank
    - 0.5, median of distances; 0.75, 75th percentile





1,1,2,2,3,3,3,3,4,4,5,12,14,15

.25 .5 .75 1.0

Hausdorff Matching

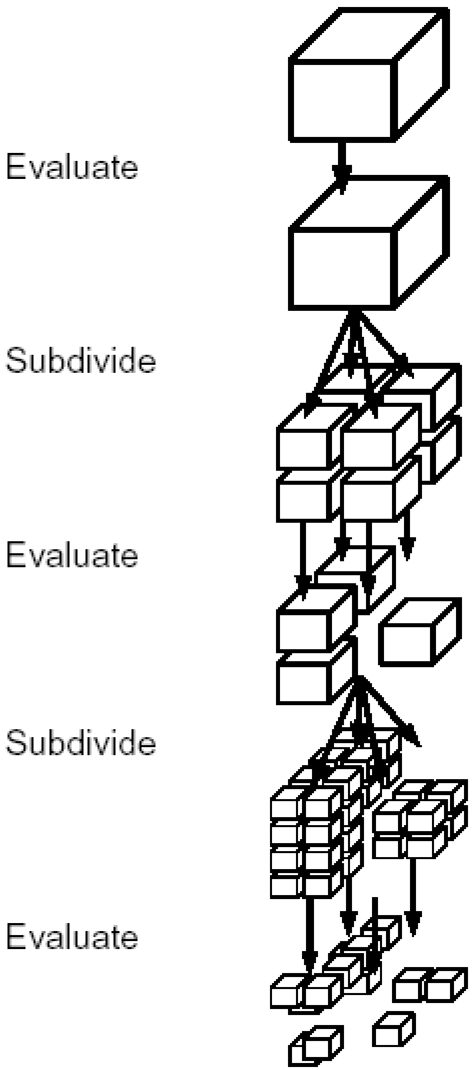
- Best match
  - Minimum fractional Hausdorff distance over given space of transformations
- Good matches
  - Above some fraction (rank) and/or below some distance
- Each point in (quantized) transformation space defines a distance
  - Search over transformation space
    - Efficient branch-and-bound “pruning” to skip transformations that cannot be good

Fast Hausdorff Search

- Branch and bound hierarchical search of transformation space
- Consider 2D transformation space of translation in x and y
  - (Fractional) Hausdorff distance cannot change faster than linearly with translation
    - Similar constraints for other transformations
  - Quad-tree decomposition, compute distance for transform at center of each cell
    - If larger than cell half-width, rule out cell
    - Otherwise subdivide cell and consider children

Branch and Bound Illustration

- Guaranteed (or admissible) search heuristic
  - Bound on how good answer could be in unexplored region
    - Cannot miss an answer
  - In worst case won't rule anything out
- In practice rule out vast majority of transformations
  - Can use even simpler tests than computing distance at cell center

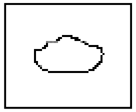


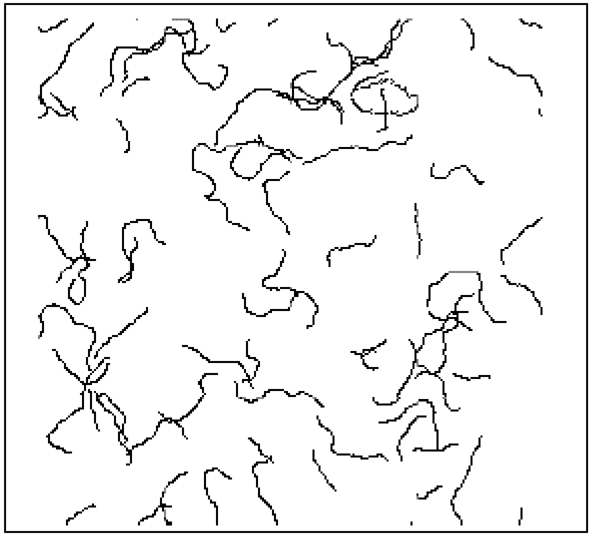
DT Based Matching Measures

- Fractional Hausdorff distance
  - Kth largest value selected from DT
- Chamfer
  - Sum of values selected from DT
    - Suffers from same robustness problems as classical Hausdorff distance
    - Max intuitively worse but sum also bad
  - Robust variants
    - Trimmed: sum the K smallest distances (same as Hausdorff but sum rather than largest of K)
    - Truncated: truncate individual distances before summing

Comparing DT Based Measures

- Monte Carlo experiments with known object location and synthetic clutter
  - Matching edge locations
- Varying percent clutter
  - Probability of edge pixel 2.5-15%
- Varying occlusion
  - Single missing interval, 10-25% of boundary
- Search over location, scale, orientation

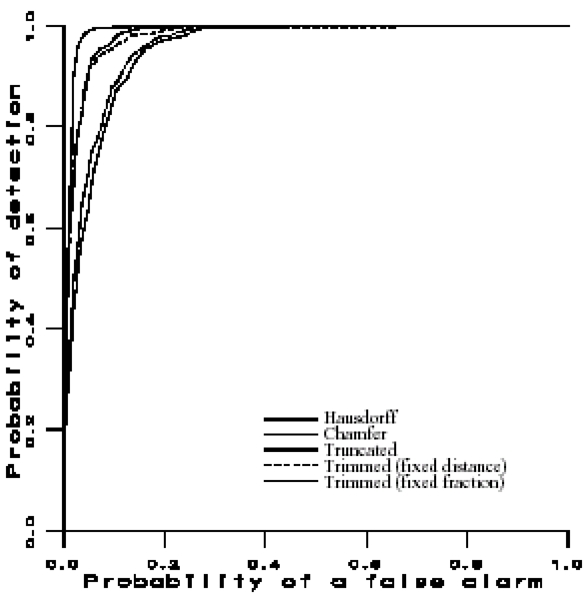


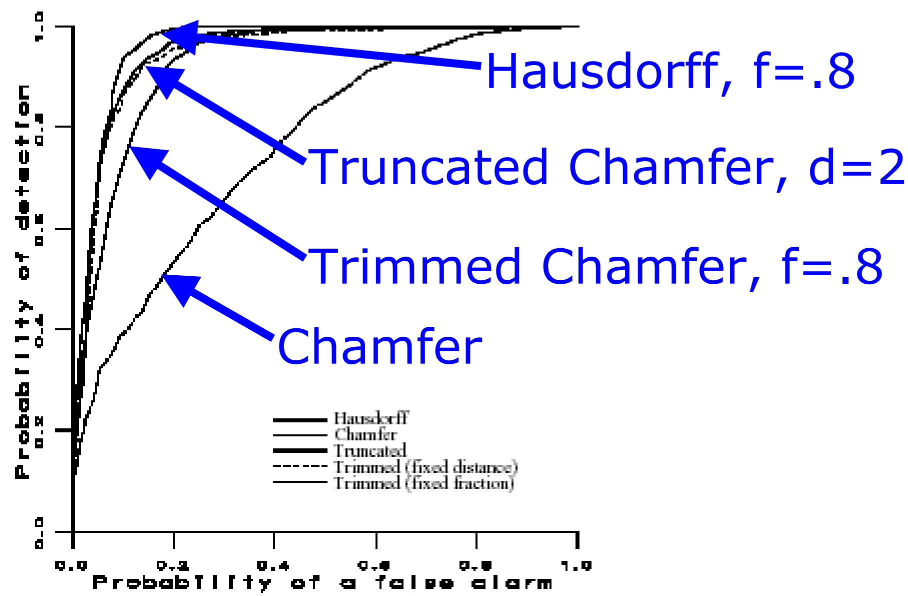


5% Clutter Image

ROC Curves

- Probability of false alarm vs. detection
  - 10% and 15% occlusion with 5% clutter
  - Chamfer is lowest, Hausdorff (f=.8) is highest
  - Chamfer truncated distance better than trimmed



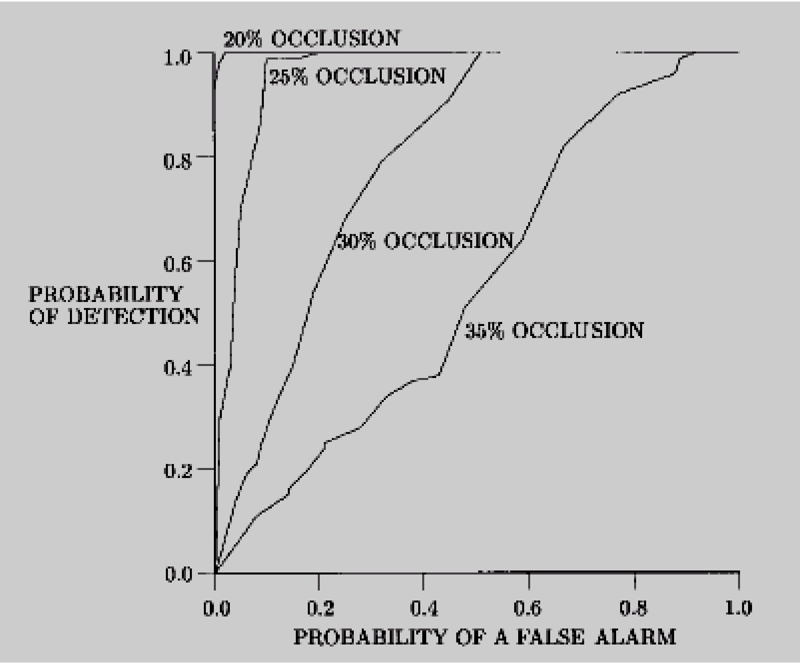


Edge Orientation Information

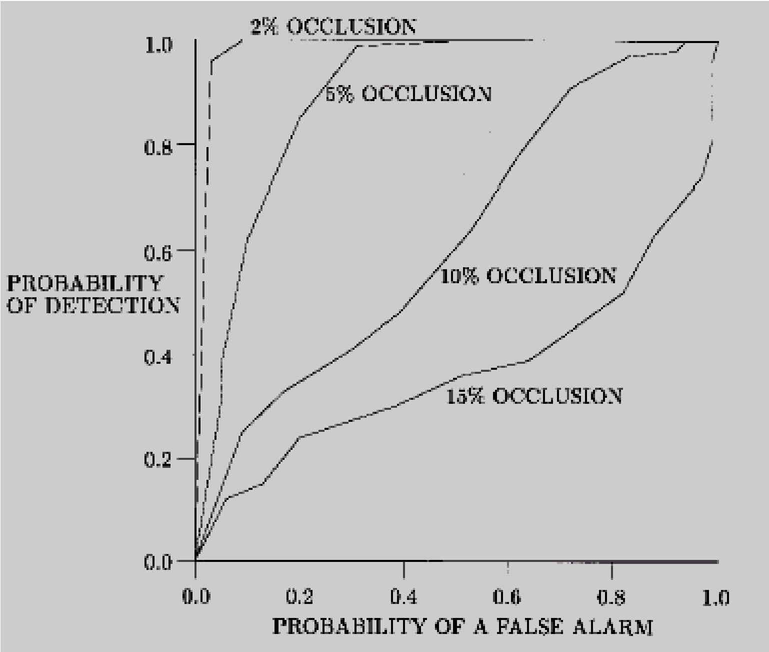
- Match edge orientation as well as location
  - Edge normals or gradient direction
- Increases detection performance and speeds up matching
  - Better able to discriminate object from clutter
  - Better able to eliminate cells in branch and bound search
- Distance in 3D feature space $[p_x, p_y, \alpha p_o]$
  - α weights orientation versus location
  - $kth_{a \in A} \min_{b \in B} \| a - b \| = kth_{a \in A} D_B(a)$

ROC's for Oriented Edge Pixels

- Vast improvement for moderate clutter
  - Images with 5% randomly generated contours
  - Good for 20-25% occlusion rather than 2-5%



Oriented Edges



Location Only

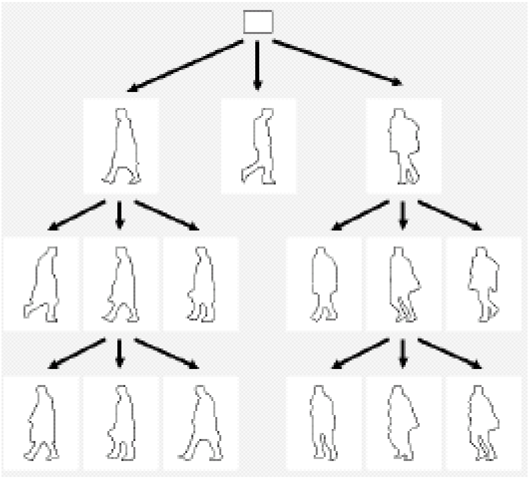
Observations on DT Based Matching

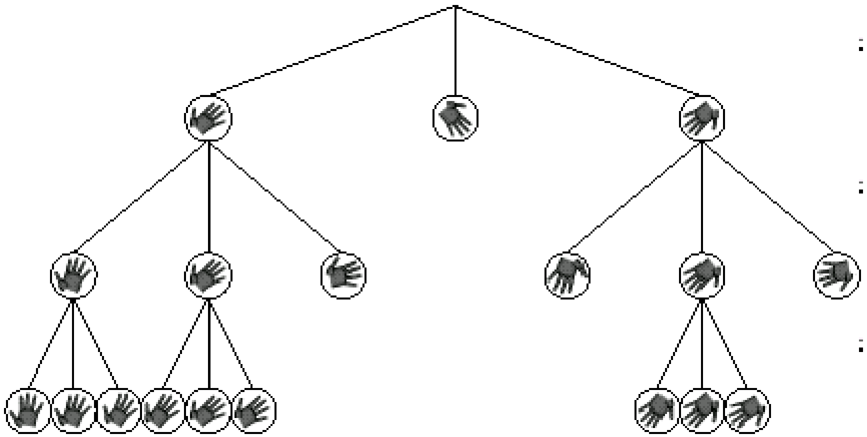
- Fast compared to explicitly considering pairs of model and data features
  - Hierarchical search over transformation space
- Important to use robust distance
  - Straight Chamfer very sensitive to outliers
    - Truncated DT can be computed fast
- No reason to use approximate DT
  - Fast exact method for L_2^2 or truncated L_2^2
- For edge features use orientation too
  - Comparing normals or using multiple edge maps

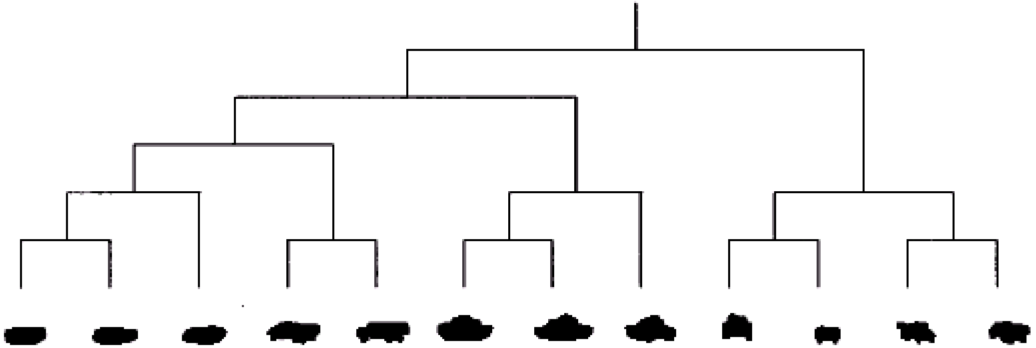
Template Clustering

- Cluster templates into tree structures to speed matching
  - Rule out multiple templates simultaneously
    - Coarse-to-fine search where coarse granularity can rule out many templates
    - Several variants: Olson, Gavrila, Stenger
- Applies to variety of DT based matching measures
  - Chamfer, Hausdorff and robust Chamfer
- Use hierarchical clustering techniques (e.g., Edelsbrunner) offline on templates

Example Hierarchical Clusters







Larger pairwise differences higher in tree

Hausdorff and Linear Halfspaces

Dilate and Correlate Matching

- Fixed degree of "smoothing" of features
  - Dilate binary feature map with specific radius disc rather than all radii as in DT
- $h_k(A,B) \leq d \iff |A \cap B^d| \geq k$
  - At least k points of A contained in B^d
- For low dimensional transformations such as x-y-translation best way to compute
  - Dilation and binary correlation are very fast
  - For higher dimensional cases hierarchical search using DT is faster

Dot Product Formulation

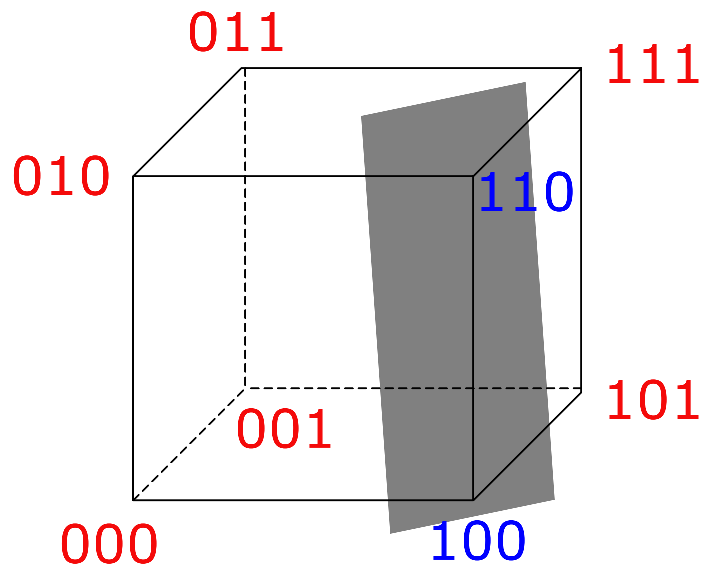
- Let $\boldsymbol{A}$ and $\boldsymbol{B}^{\mathbf{d}}$ be (binary) vector representations of A and B
  - E.g. standard scan line order
- Then fractional Hausdorff distance can be expressed as dot product
  - $h_{\mathbf{k}}(A,B) \leq d \Leftrightarrow \boldsymbol{A} \bullet \boldsymbol{B}^{\mathbf{d}} \geq k$
- Note that if B is perturbation of A by d then $\boldsymbol{A} \bullet \boldsymbol{B}$ is arbitrary whereas $\boldsymbol{A} \bullet \boldsymbol{B}^{\mathbf{d}} = \boldsymbol{A} \bullet \boldsymbol{A}$
- Hausdorff matching using linear subspaces
  - Eigenspace, PCA, etc.

Learning and Hausdorff Distance

- Learning linear half spaces
  - Dot product formulation defines linear threshold function
    - Positive if $\boldsymbol{A} \bullet \boldsymbol{B}^{\mathbf{d}} \geq k$, negative otherwise
- PAC – probably approximately correct
  - Learning concepts that with high probability have low error
  - Linear programming and perceptrons can both be used to learn half spaces in PAC sense
- Consider small number of values for d (dilation parameter) and pick best

Illustration of Linear Halfspace

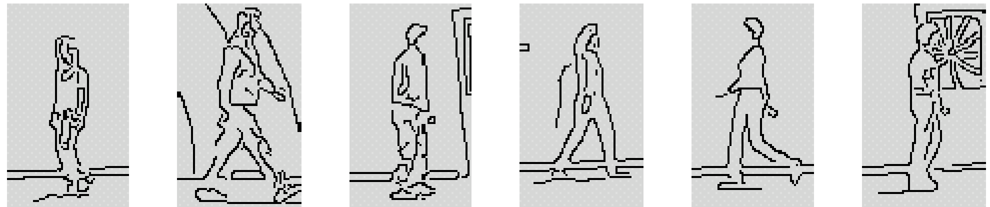
- Possible images define n-dimensional binary space
- Linear function separating positive and negative examples



Perceptron Algorithm

- Examples x_i each with label $y_i \in \{+,-\}$
- Set initial prediction vector v to 0
- For i=1, …, m
  - If $\text{sign}(v \bullet x_i) \neq \text{sign}(y_i)$ then $v = v + y_i x_i$
- Run repeatedly until no misclassifications on m training examples
  - Or less than some threshold number but then haven't found linear separator
- Generally need many more negative than positive examples for effective training

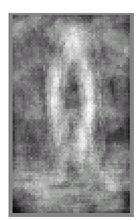
Learned Half-Space Templates



Positive examples (500)

Negative examples (350,000)

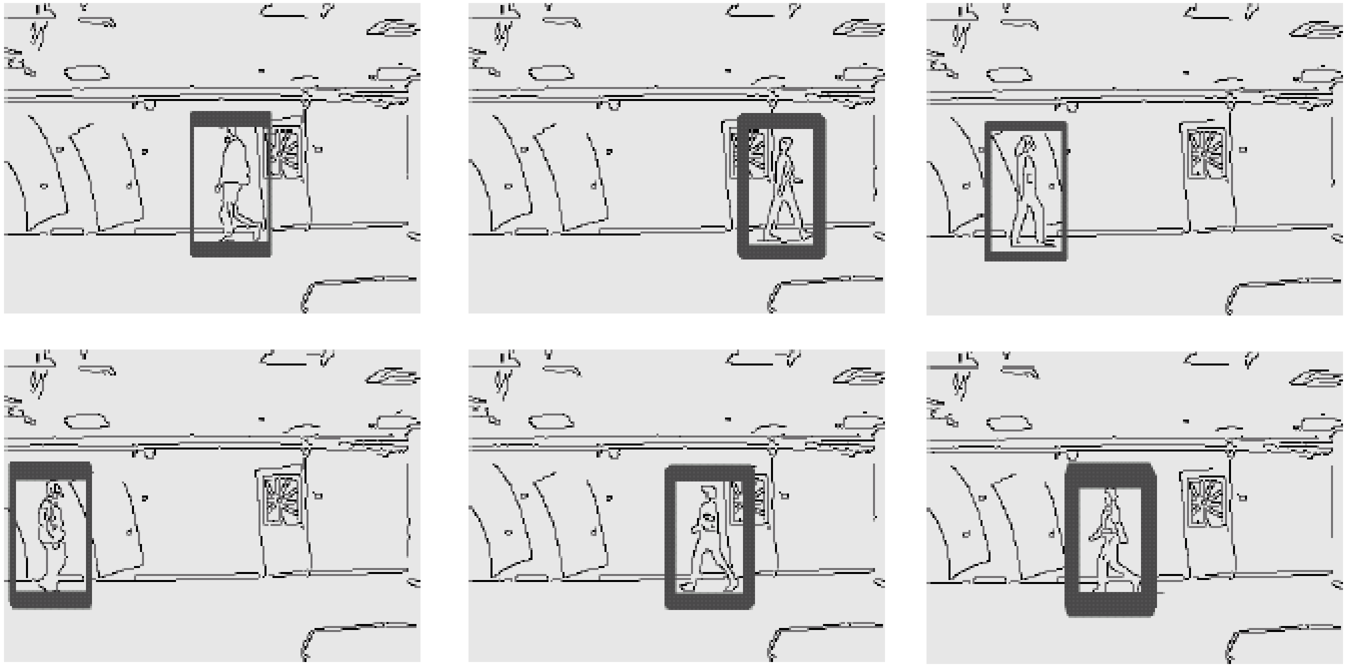
All Model Coefs.



Pos. Model Coefs.

Example Model (dilation d=3, picked automatically)

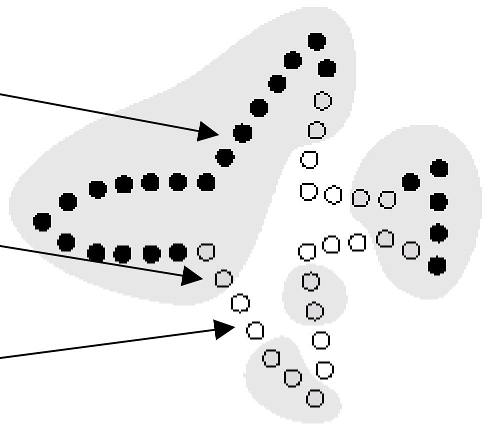
Detection Results



- Train on 80% test on 20% of data
  - No trials yielded any false positives
  - Average 3% missed detections, worst case 5%

Spatial Continuity

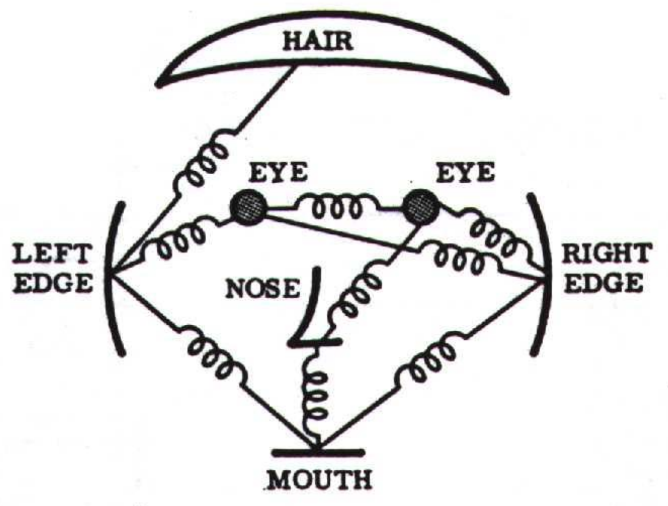
- Hausdorff and Chamfer matching do not measure degree of connectivity
  - E.g., edge chains versus isolated points
- Spatially coherent matching approach
  - Separate features into three subsets
    - Matchable
      - Near image features
    - Boundary
      - Matchable but near un-matchable
    - Un-matchable
      - Far from image features



Flexible Templates

Flexible Template Matching

- Pictorial structures
  - Parts connected by springs and appearance models for each part
  - Used for human bodies, faces
  - Fischler&Elschlager, 1973 – considerable recent work

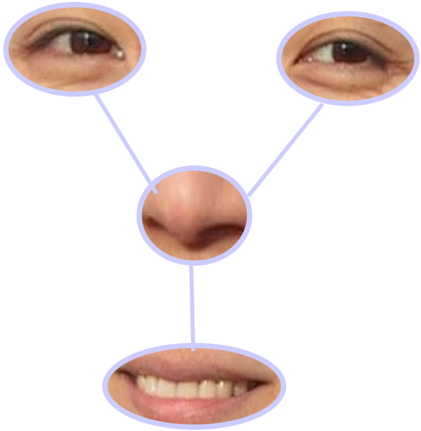


Formal Definition of Model

- Set of parts $V=\{v_1, \ldots, v_n\}$
- Configuration $L=(l_1, \ldots, l_n)$
  - Specifying locations of the parts
- Appearance parameters $A=(a_1, \ldots, a_n)$
  - Model for each part
- Edge e_{ij}, $(v_i,v_j) \in E$ for connected parts
  - Explicit dependency between part locations l_i, l_j
- Connection parameters $C=\{c_{ij} \mid e_{ij} \in E\}$
  - Spring parameters for each pair of connected parts

Flexible Template Algorithms

- Difficulty depends on structure of graph
  - Which parts are connected (E) and how (C)
- General case exponential time
  - Consider special case in which parts translate with respect to common origin
    - E.g., useful for faces



- Distinguished central part v_1
- Spring c_{i1} connecting v_i to v_1
- Quadratic cost for spring

Efficient Algorithm for Central Part

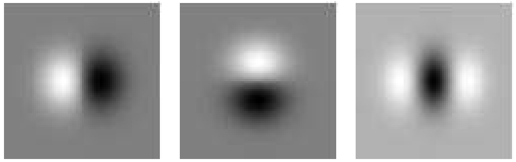
- Location $L=(l_1, \ldots, l_n)$ specifies where each part positioned in image
- Best location $\min_L (\Sigma_i\, m_i(l_i) + d_i(l_i,l_1))$
  - Part cost $m_i(l_i)$
    - Measures degree of mismatch of appearance a_i when part v_i placed at location l_i
  - Deformation cost $d_i(l_i,l_1)$
    - Spring cost c_{i1} of part v_i measured with respect to central part v_1
    - E.g., quadratic or truncated quadratic function
    - Note deformation cost zero for part v_1 (wrt self)

Express as Kind of DT

- $\min_L (\Sigma_i (m_i(l_i) + d_i(l_i,l_1)))$
- $\min_L (\Sigma_i m_i(l_i) + \| l_i - T_i(l_1) \|^2)$
  - Quadratic distance between location of part v_i and ideal location given location of central part
- $\min_{l1} (m_1(l_1) +$
  $\Sigma_{i>1} \min_{li} (m_i(l_i) + \| l_i - T_i(l_1) \|^2))$
  - i-th term of sum minimizes only over l_i
- $\min_{l1} (m_1(l_1) + \Sigma_{i>1} D_{mi}(T_i(l_1)))$
  - Each term of sum is <u>distance transform</u> of the match cost function m_i
    - $D_f(x) = \min_y (f(y) + \| y-x \|^2)$, using same algorithms as before

Application to Face Detection

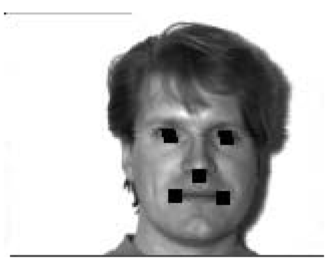
- Five parts: eyes, tip of nose, sides of mouth
- Each part a local image patch (m_i)
  - Represented as response to oriented filters

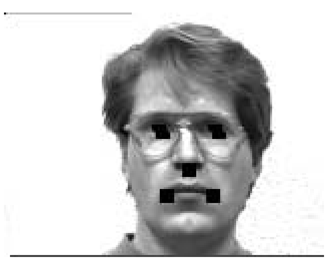


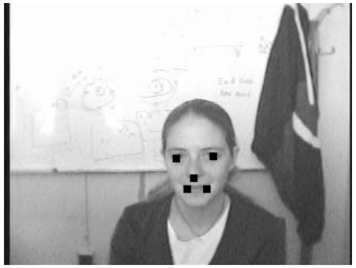
  - 27 filters at 3 scales and 9 orientations
  - Learn coefficients from labeled examples
- Parts translate with respect to central part, tip of nose (d_i)

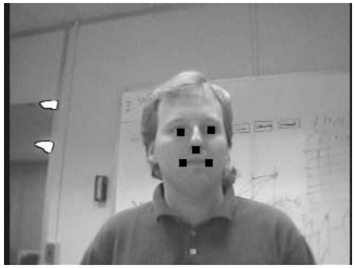
Flexible Template Face Detection

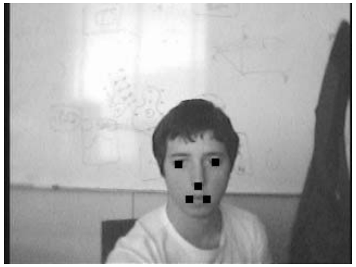
- Runs at several frames per second
  - Compute oriented filters at 27 orientations and scales for part cost m_i
  - Distance transform m_i for each part other than central one (nose tip)
  - Find maximum of sum for detected location

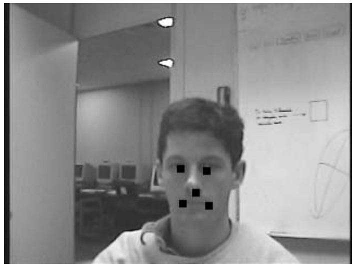












More General Flexible Templates

- Efficient computation using distance transforms for any tree-structured model
  - Not limited to central reference part
- Two differences from reference part case
  - Relate positions of parts to one another using tree-structured recursion
    - Solve with Viterbi or forward-backward algorithm
  - Parameterization of distance transform more complex – transformation T_{ij} for each connected pair of parts

General Form of Problem

- Best location can be viewed in terms of probability or cost (negative log prob.)
  - $\max_L p(L|I,\Theta) = \arg\max_L p(I|L,A)p(L|E,C)$
  - $\min_L \Sigma_V m_j(l_j) + \Sigma_E d_{ij}(l_i,l_j)$
    - $m_j(l_j)$ – how well part v_j matches image at l_j
    - $d_{ij}(l_i,l_j)$ – how well locations l_i,l_j agree with model (spring connecting parts v_i and v_j)
- Difficulty of maximization/minimization depends on form of graph
  - Exponential time in general, efficient for tree

Minimizing Over Tree Structures

- Use dynamic programming to minimize $\Sigma_V m_j(l_j) + \Sigma_E d_{ij}(l_i,l_j)$
- Can express as function for pairs $B_j(l_i)$
  - Cost of best location of v_j given location l_i of v_i
- Recursive formulas in terms of children C_j of v_j
  - $B_j(l_i) = \min_{l_j} (m_j(l_j) + d_{ij}(l_i,l_j) + \Sigma_{Cj} B_c(l_j))$
  - For leaf node no children, so last term empty
  - For root node no parent, so second term omitted

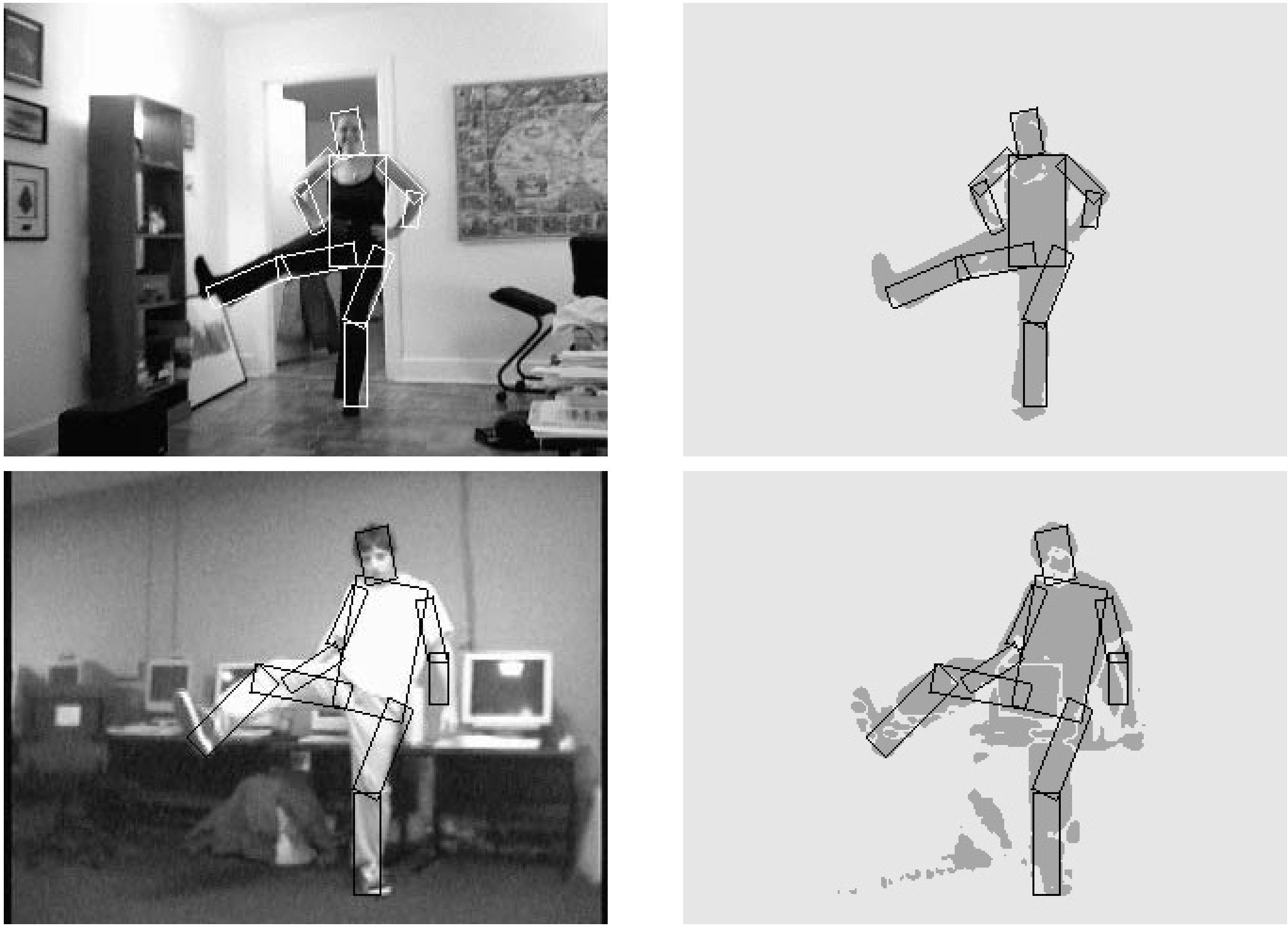
Efficient Algorithm for Trees

- MAP estimation algorithm
  - Tree structure allows use of Viterbi style dynamic programming
    - $O(ns^2)$ rather than $O(s^n)$ for s locations, n parts
    - Still slow to be useful in practice (s in millions)
  - Couple with distance transform method for finding best pair-wise locations in linear time
    - Resulting O(ns) method
- Similar techniques allow sampling from posterior distribution in O(ns) time
  - Using forward-backward algorithm

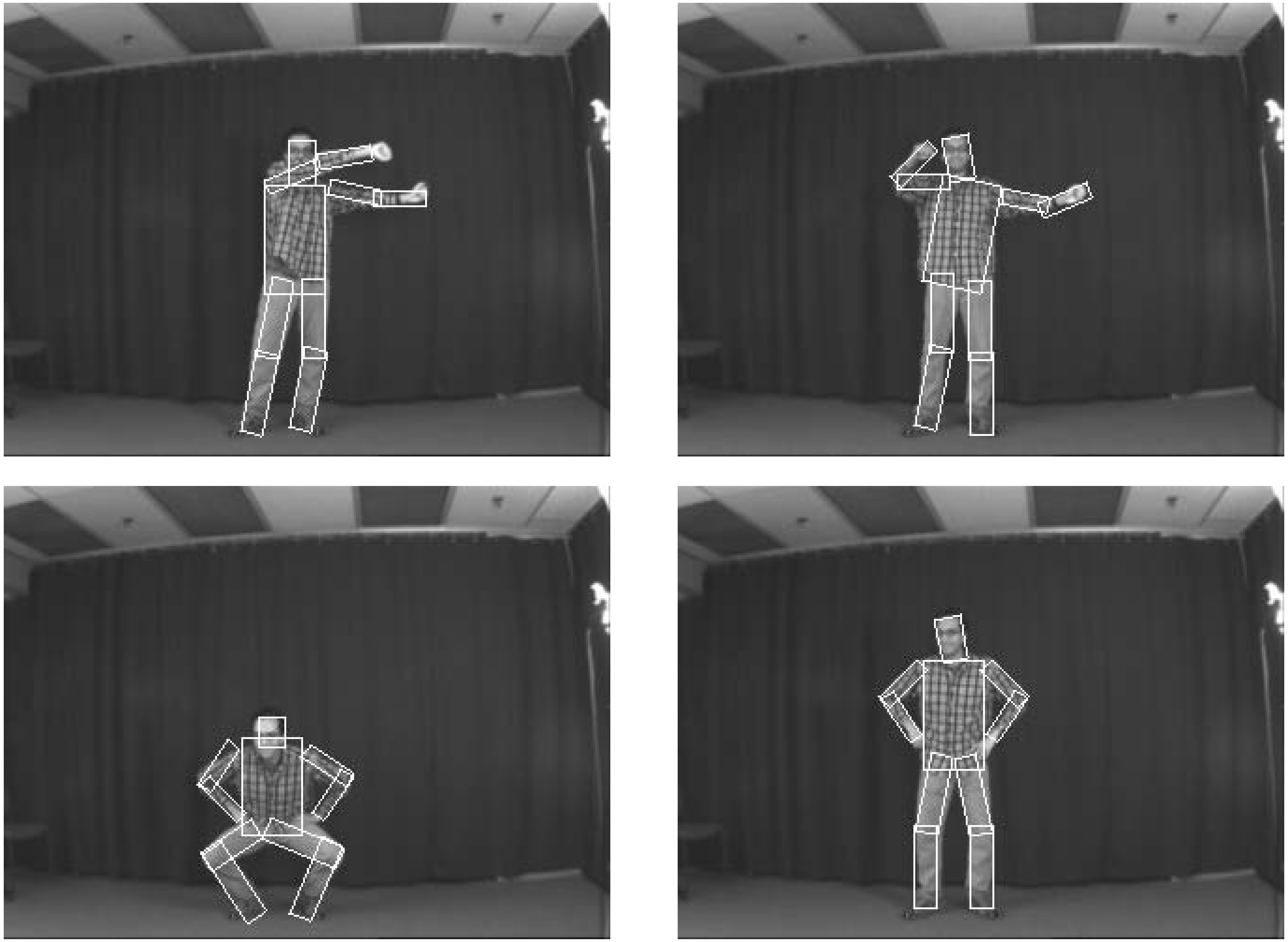
O(ns) Algorithm for MAP Estimate

- Express $B_j(l_i)$ in recursive minimization formulas as a DT $D_f(T_{ij}(l_i))$
  - Cost function
    - $f(y) = m_{\mathbf{j}}(T_{\mathbf{ji}}^{-1}(y)) + \sum_{\mathbf{Cj}} B_{\mathbf{c}}(T_{\mathbf{ji}}^{-1}(y))$
  - $T_{\mathbf{ij}}, T_{\mathbf{ji}}$ map locations to space where difference between $l_{\mathbf{i}}$ and $l_{\mathbf{j}}$ is a squared distance
    - Distance zero at ideal relative locations
- Yields n recursive equations
  - Each can be computed in O(sD) time
    - D is number of dimensions to parameter space but is fixed (in our case D is 2 to 4)

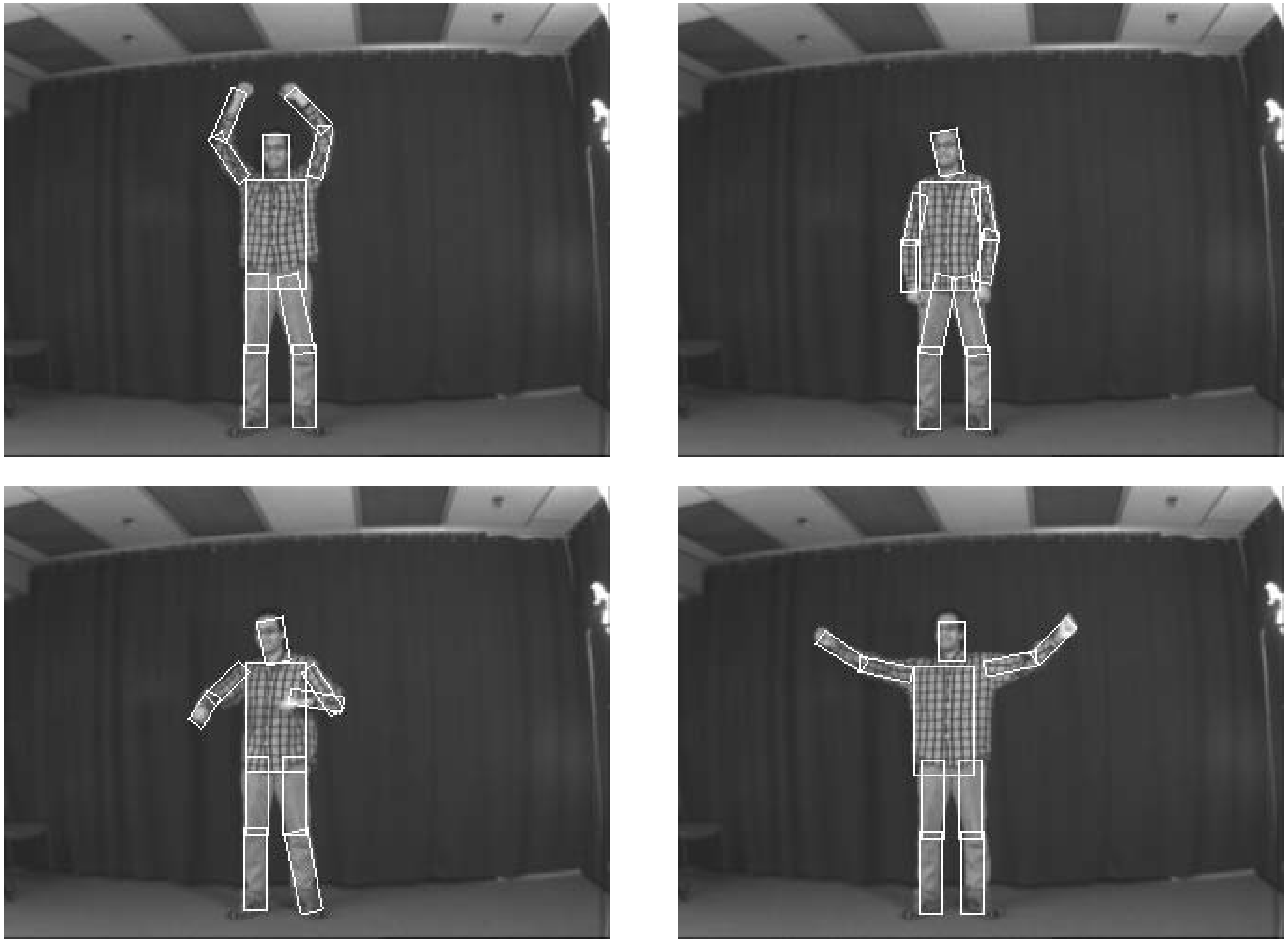
Example: Recognizing People



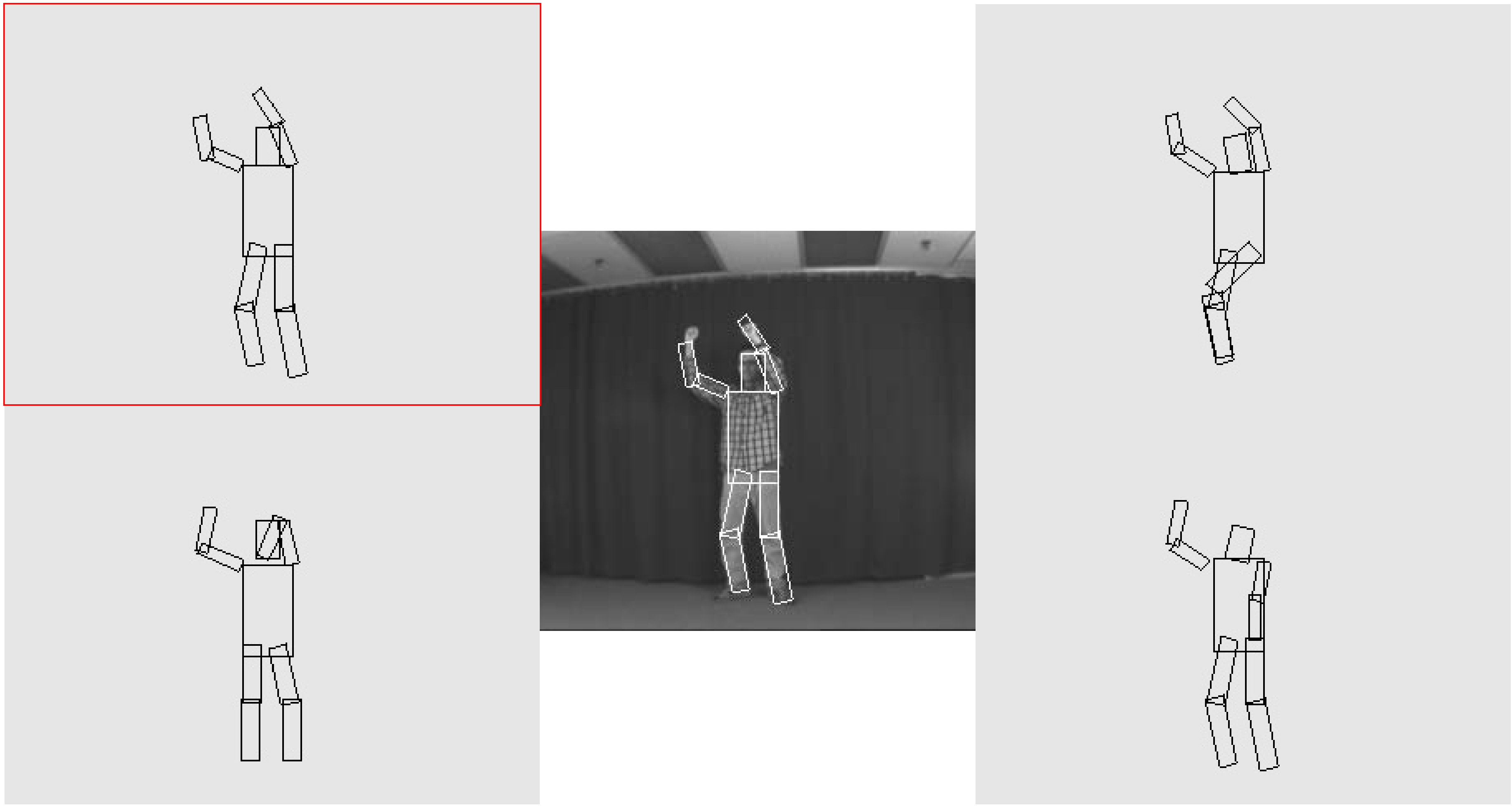
Variety of Poses



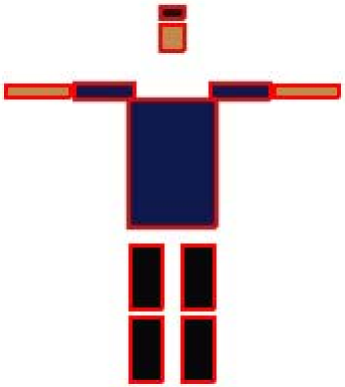
Variety of Poses

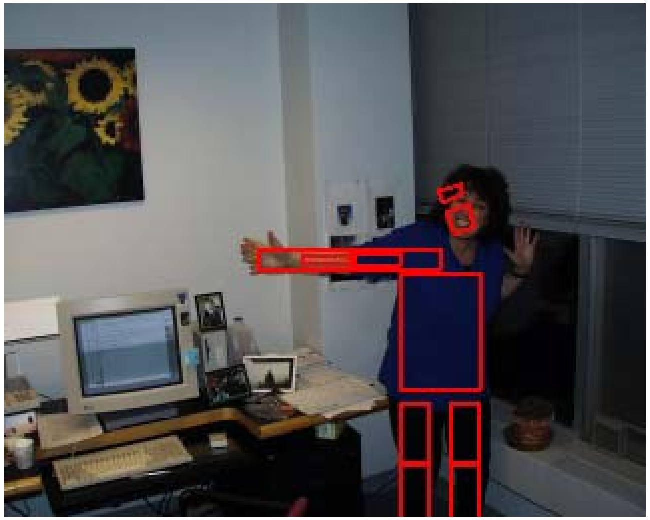


Samples From Posterior



Model of Specific Person





Bayesian Formulation of Learning

- Given example images $I^1, \ldots, I^m$ with configurations $L^1, \ldots, L^m$
  - Supervised or labeled learning problem
- Obtain estimates for model $\Theta=(A,E,C)$
- Maximum likelihood (ML) estimate is
  - $\text{argmax}_\Theta \; p(I^{\mathbf{1}}, \ldots, I^{\mathbf{m}}, L^{\mathbf{1}}, \ldots, L^{\mathbf{m}} \mid \Theta)$
  - $\text{argmax}_\Theta \; \prod_{\mathbf{k}} p(I^{\mathbf{k}},L^{\mathbf{k}}|\Theta)$
    - Independent examples
  - $\text{argmax}_\Theta \; \prod_{\mathbf{k}} p(I^{\mathbf{k}}|L^{\mathbf{k}},A) \; \prod_{\mathbf{k}} p(L^{\mathbf{k}}|E,C)$
    - Independent appearance and dependencies

Efficiently Learning Tree Models

- Estimating appearance $p(I^k|L^k,A)$
  - ML estimation for particular type of part
    - E.g., for constant color patch use Gaussian model, computing mean color and covariance
- Estimating dependencies $p(L^k|E,C)$
  - Estimate C for pairwise locations, $p(l_i^k,l_j^k|c_{ij})$
    - E.g., for translation compute mean offset between parts and variation in offset
  - Best tree using minimum spanning tree (MST) algorithm
    - Pairs with "smallest relative spatial variation"

Example: Generic Person Model

- Each part represented as rectangle
  - Fixed width, varying length
  - Learn average and variation
    - Connections approximate revolute joints
  - Joint location, relative position, orientation, foreshortening
  - Estimate average and variation
- Learned model (used above)
  - All parameters learned
    - Including "joint locations"
  - Shown at ideal configuration