

# Instance-level recognition: Local invariant features

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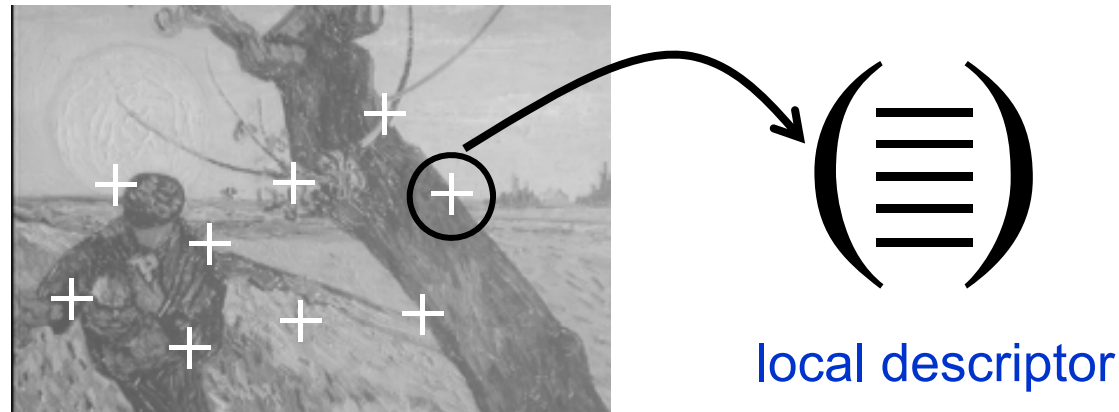
# Overview

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- **Introduction to local features**
- Harris interest points + SSD, ZNCC, SIFT
- Scale & affine invariant interest point detectors
- Evaluation and comparison of different detectors
- Region descriptors and their performance

# Local features

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Several / many local descriptors per image

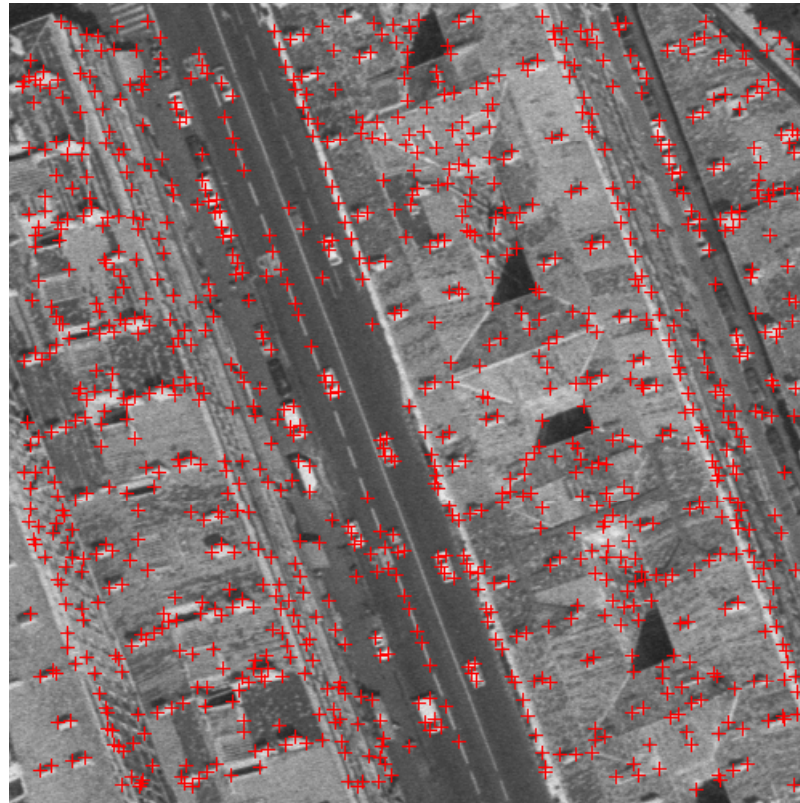
Robust to occlusion/clutter + no object segmentation required

*Photometric* : distinctive

*Invariant* : to image transformations + illumination changes

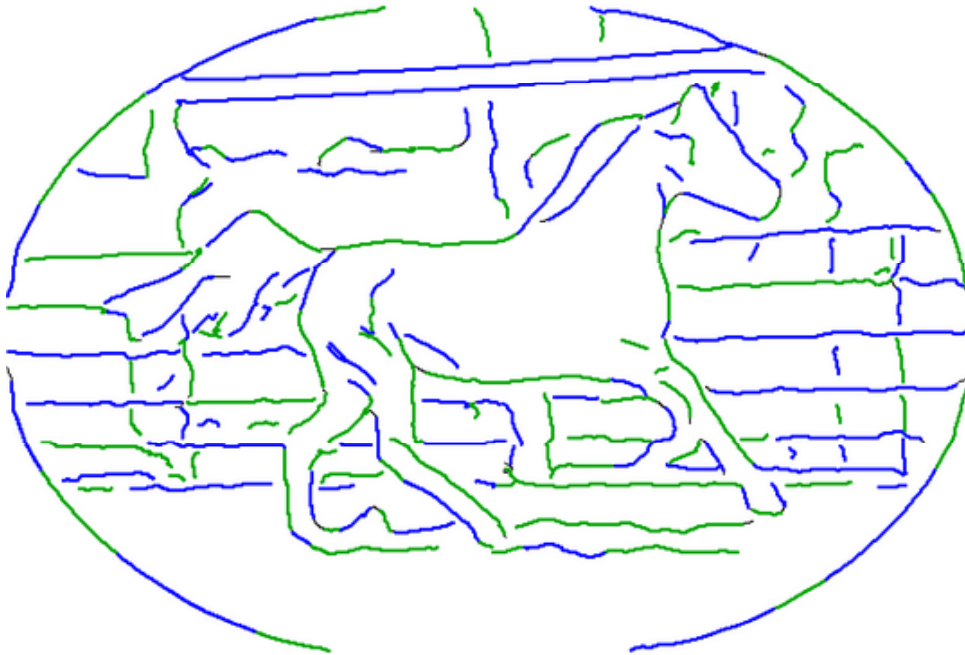
# Local features: interest points

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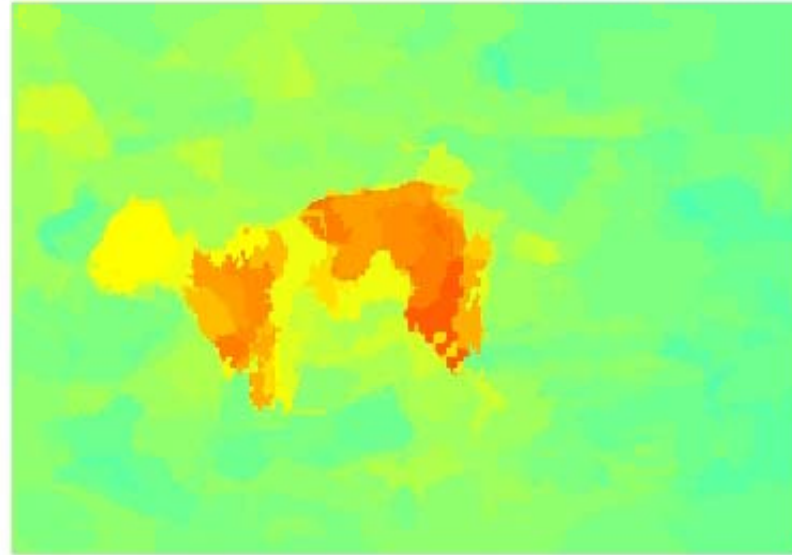
# Local features: Contours/segments

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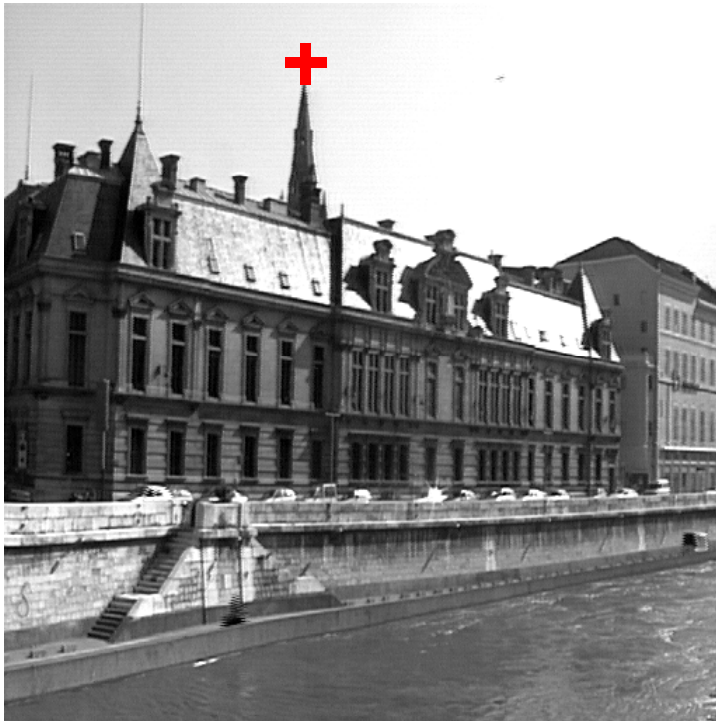
# Local features: segmentation

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# Application: Matching

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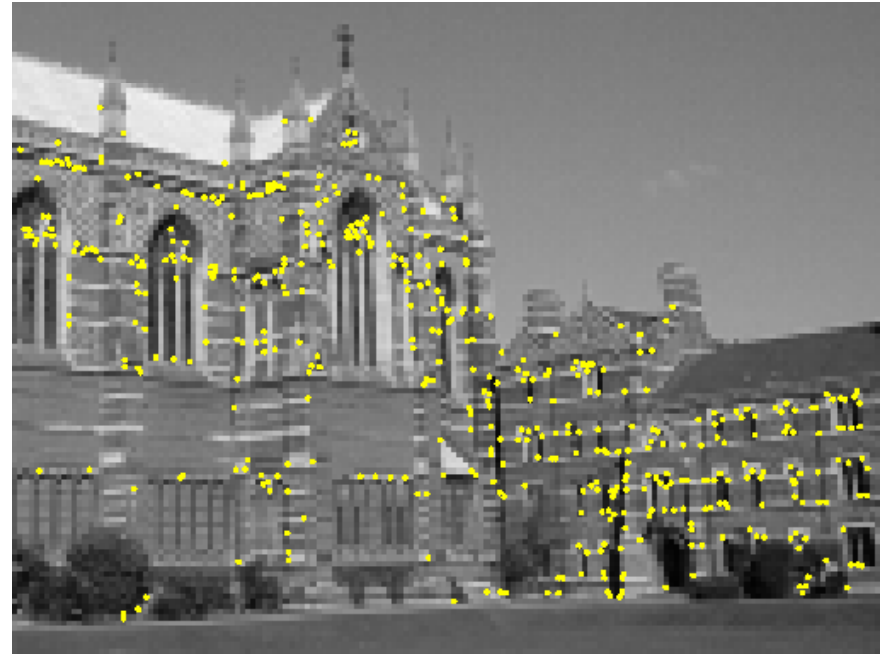
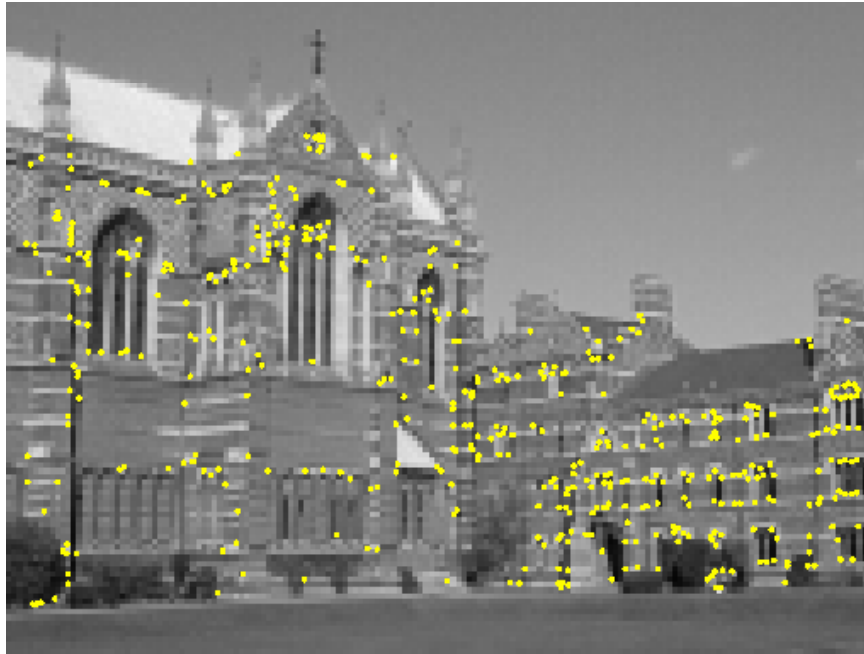


Find corresponding locations in the image



# Illustration – Matching

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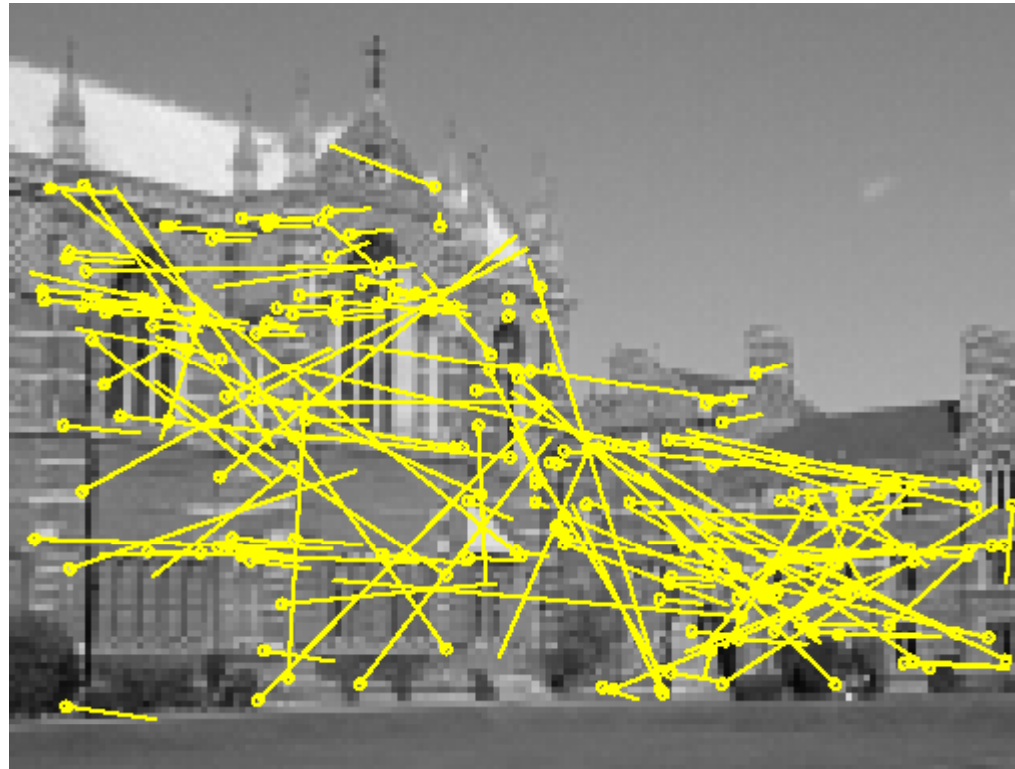


Interest points extracted with Harris detector (~ 500 points)



# Illustration – Matching

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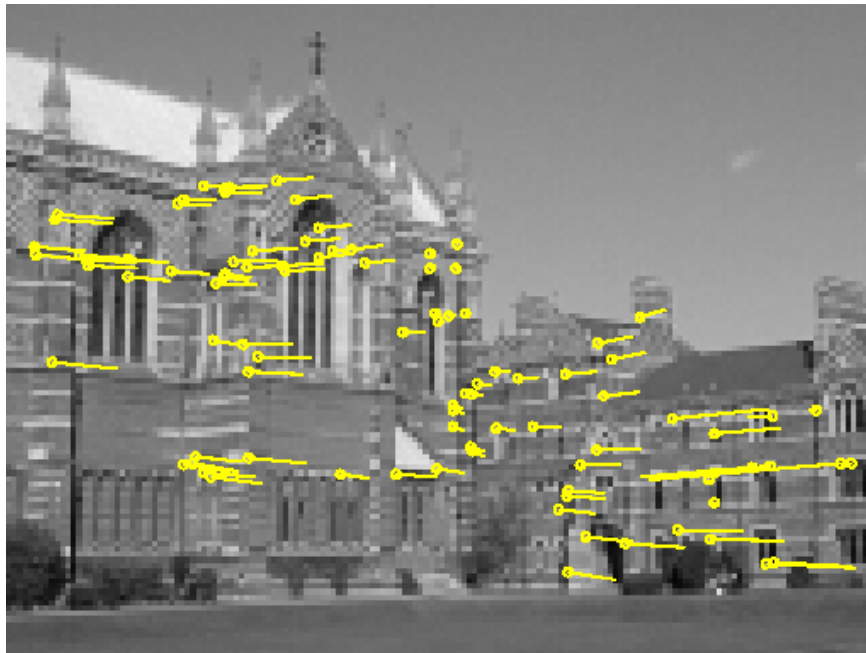


Interest points matched based on cross-correlation (188 pairs)

# Illustration – Matching

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Global constraint - Robust estimation of the fundamental matrix



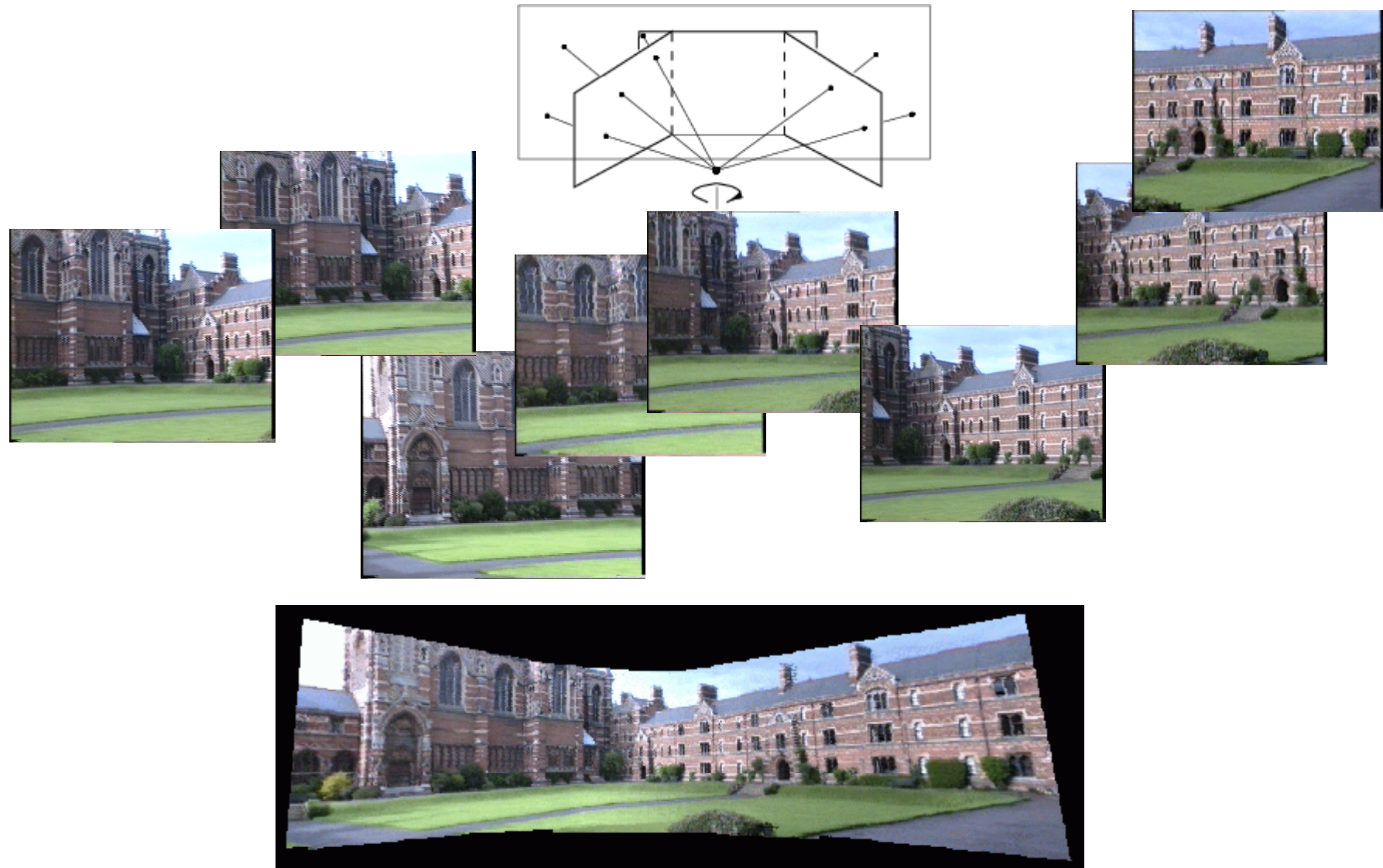
99 inliers



89 outliers

# Application: Panorama stitching

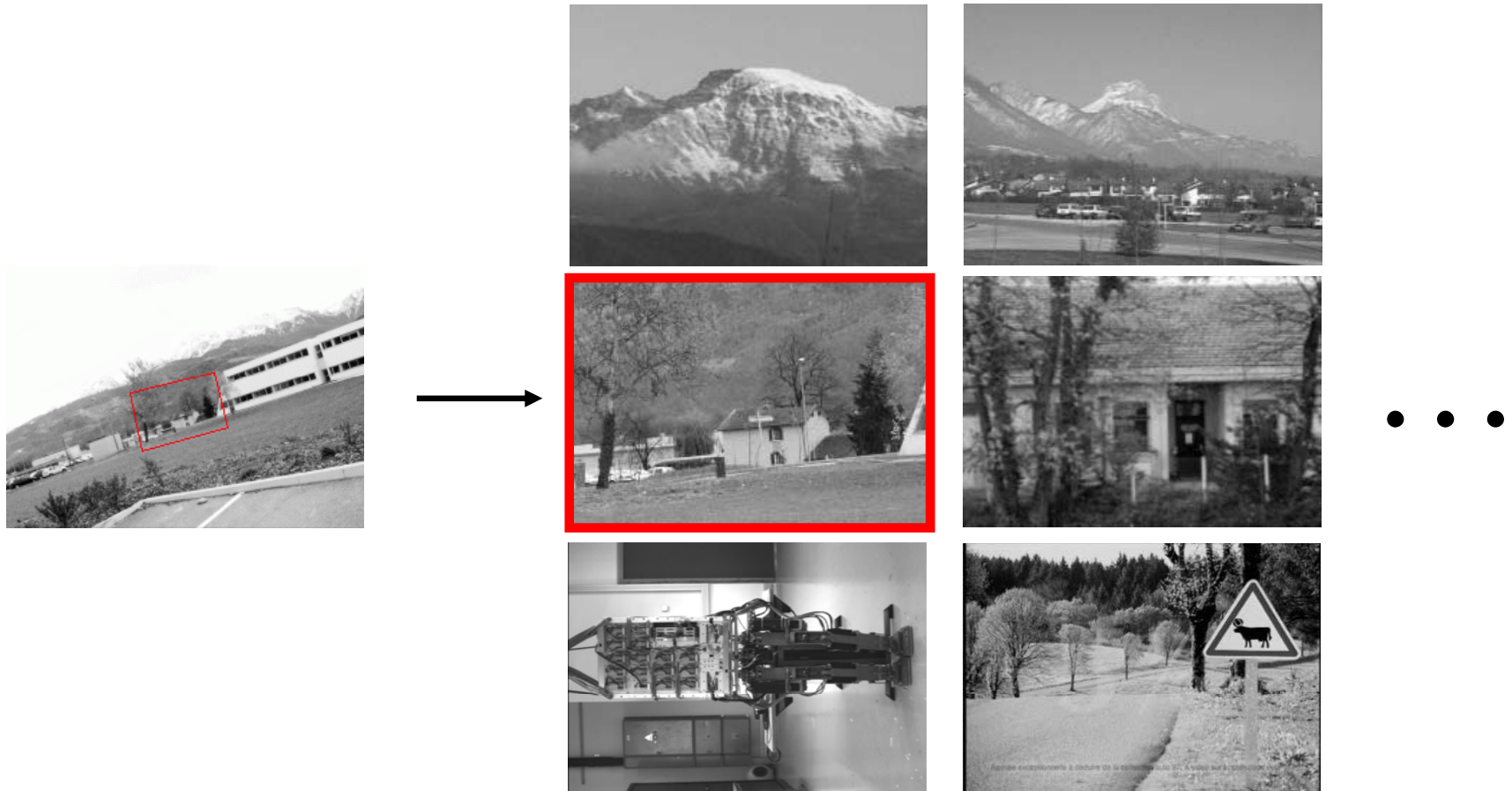
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# Application: Instance-level recognition

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Search for particular objects and scenes in large databases



# Difficulties

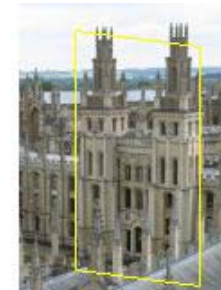
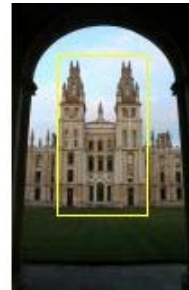
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Finding the object despite possibly large changes in scale, viewpoint, lighting and partial occlusion

→ **requires invariant description**



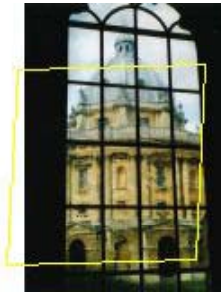
Scale



Viewpoint



Lighting



Occlusion



# Difficulties

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- Very large images collection → need for efficient indexing
  - Flickr has 2 billion photographs, more than 1 million added daily
  - Facebook has 15 billion images (~27 million added daily)
  - Large personal collections
  - Video collections, i.e., YouTube





# Applications

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- Take a picture of a product or advertisement  
→ find relevant information on the web

## PRENEZ EN PHOTO L'AFFICHE !

Accédez à la bande annonce, à tous les horaires et à la réservation.

Avec la participation de



TOUTLECIINE.COM

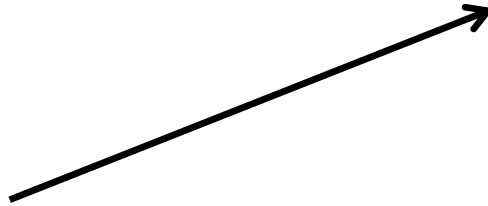


[Pixee – Milpix]

# Applications

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- Finding stolen/missing objects in a large collection

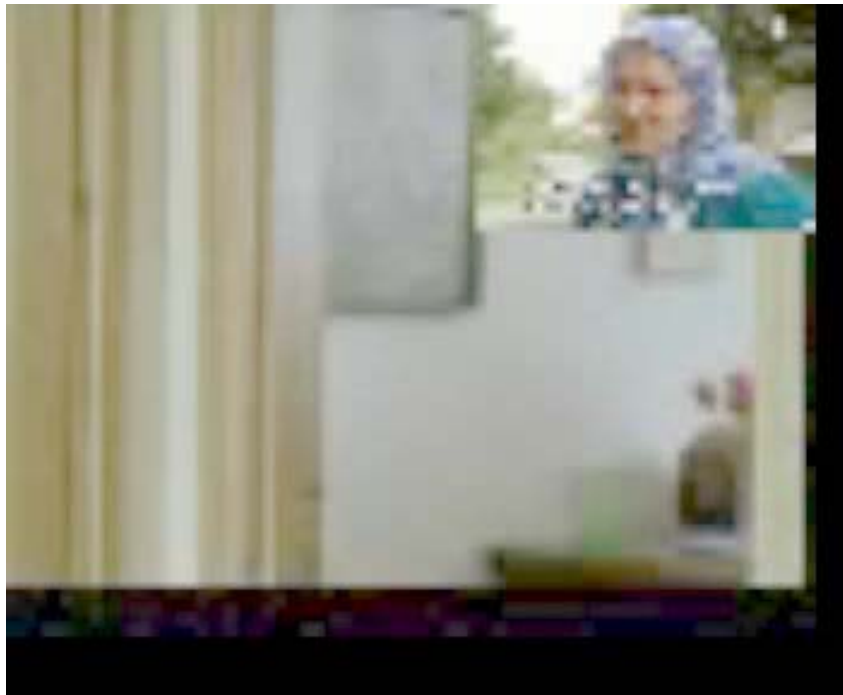


# Applications

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- Copy detection for images and videos

Query video



Search in 200h of video



# Applications

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- Sony Aibo – Robotics
  - Recognize docking station
  - Communicate with visual cards
  - Place recognition
  - Loop closure in SLAM



# Local features

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## 1) Extraction of local features

- Contours/segments
- Interest points & regions
- Regions by segmentation
- Dense features, points on a regular grid

## 2) Description of local features

- Dependant on the feature type
- Contours/segments → angles, length ratios
- Interest points → greylevels, gradient histograms
- Regions (segmentation) → texture + color distributions

# Line matching

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- Extraction de contours
  - Zero crossing of Laplacian
  - Local maxima of gradients
- Chain contour points (hysteresis)
- Extraction of line segments
- Description of segments
  - Mi-point, length, orientation, angle between pairs etc.

# Experimental results – line segments

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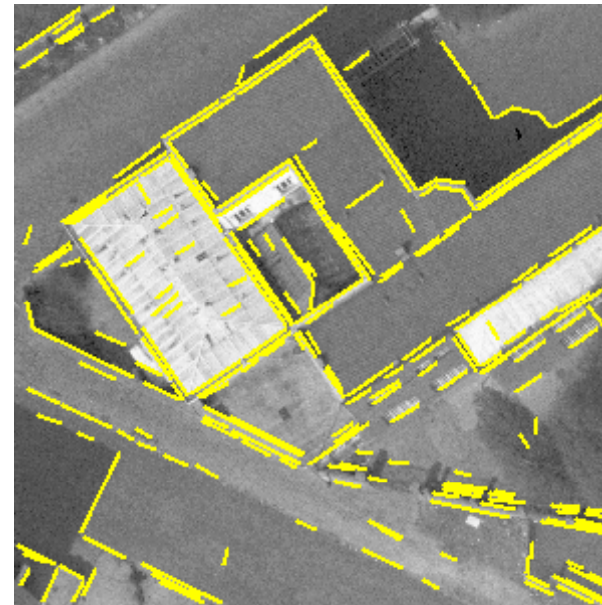


images 600 x 600



# Experimental results – line segments

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248 / 212 line segments extracted

# Experimental results – line segments

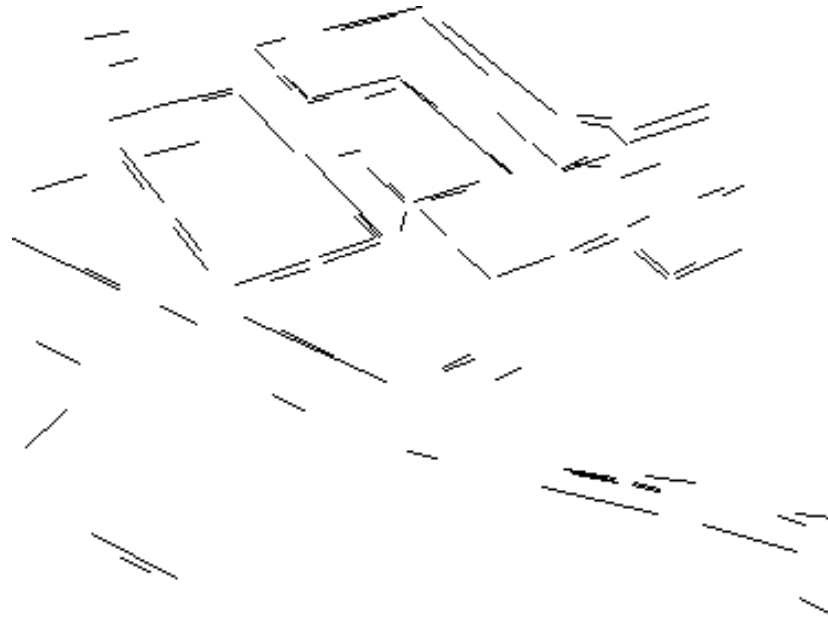
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89 matched line segments - 100% correct

# Experimental results – line segments

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3D reconstruction

# Problems of line segments

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- Often only partial extraction
  - Line segments broken into parts
  - Missing parts
- Information not very discriminative
  - 1D information
  - Similar for many segments
- Potential solutions
  - Pairs and triplets of segments
  - Interest points

# Overview

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- Introduction to local features
- **Harris interest points + SSD, ZNCC, SIFT**
- Scale & affine invariant interest point detectors
- Evaluation and comparison of different detectors
- Region descriptors and their performance

# Harris detector [Harris & Stephens'88]

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Based on the idea of auto-correlation



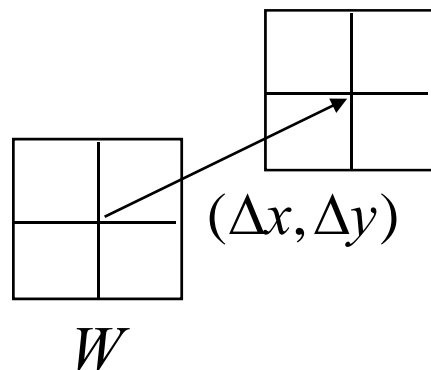
Important difference in all directions => interest point

# Harris detector

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Auto-correlation function for a point  $(x, y)$  and a shift  $(\Delta x, \Delta y)$

$$A(x, y) = \sum_{(x_k, y_k) \in W(x, y)} (I(x_k, y_k) - I(x_k + \Delta x, y_k + \Delta y))^2$$



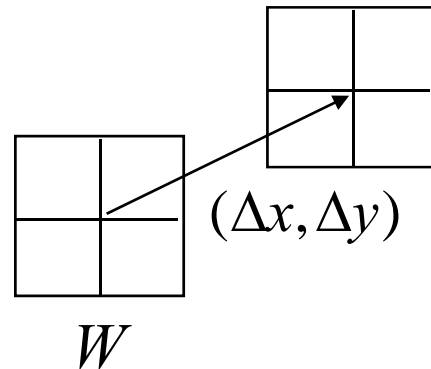


# Harris detector

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Auto-correlation function for a point  $(x, y)$  and a shift  $(\Delta x, \Delta y)$

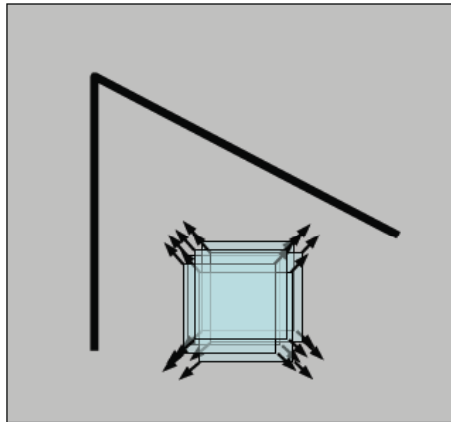
$$A(x, y) = \sum_{(x_k, y_k) \in W(x, y)} (I(x_k, y_k) - I(x_k + \Delta x, y_k + \Delta y))^2$$



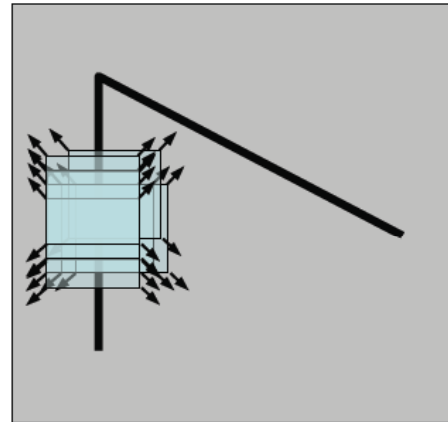
$A(x, y)$  {  
small in all directions → uniform region  
large in one directions → contour  
large in all directions → interest point

# Harris detector

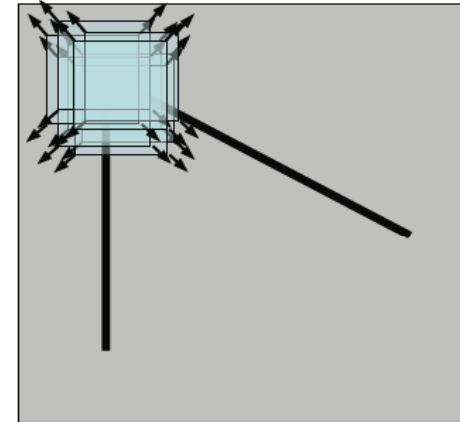
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“flat” region:  
no change in  
all directions



“edge”:  
no change along  
the edge direction



“corner”:  
significant change  
in all directions

# Harris detector

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Discret shifts are avoided based on the auto-correlation matrix

with first order approximation

$$I(x_k + \Delta x, y_k + \Delta y) = I(x_k, y_k) + \begin{pmatrix} I_x(x_k, y_k) & I_y(x_k, y_k) \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

$$\begin{aligned} A(x, y) &= \sum_{(x_k, y_k) \in W(x, y)} (I(x_k, y_k) - I(x_k + \Delta x, y_k + \Delta y))^2 \\ &= \sum_{(x_k, y_k) \in W} \left( \begin{pmatrix} I_x(x_k, y_k) & I_y(x_k, y_k) \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \right)^2 \end{aligned}$$

# Harris detector

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$$= \begin{pmatrix} \Delta x & \Delta y \end{pmatrix} \begin{bmatrix} \sum_{(x_k, y_k) \in W} (I_x(x_k, y_k))^2 & \sum_{(x_k, y_k) \in W} I_x(x_k, y_k) I_y(x_k, y_k) \\ \sum_{(x_k, y_k) \in W} I_x(x_k, y_k) I_y(x_k, y_k) & \sum_{(x_k, y_k) \in W} (I_y(x_k, y_k))^2 \end{bmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

Auto-correlation matrix

the sum can be smoothed with a Gaussian

$$= \begin{pmatrix} \Delta x & \Delta y \end{pmatrix} G \otimes \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

# Harris detector

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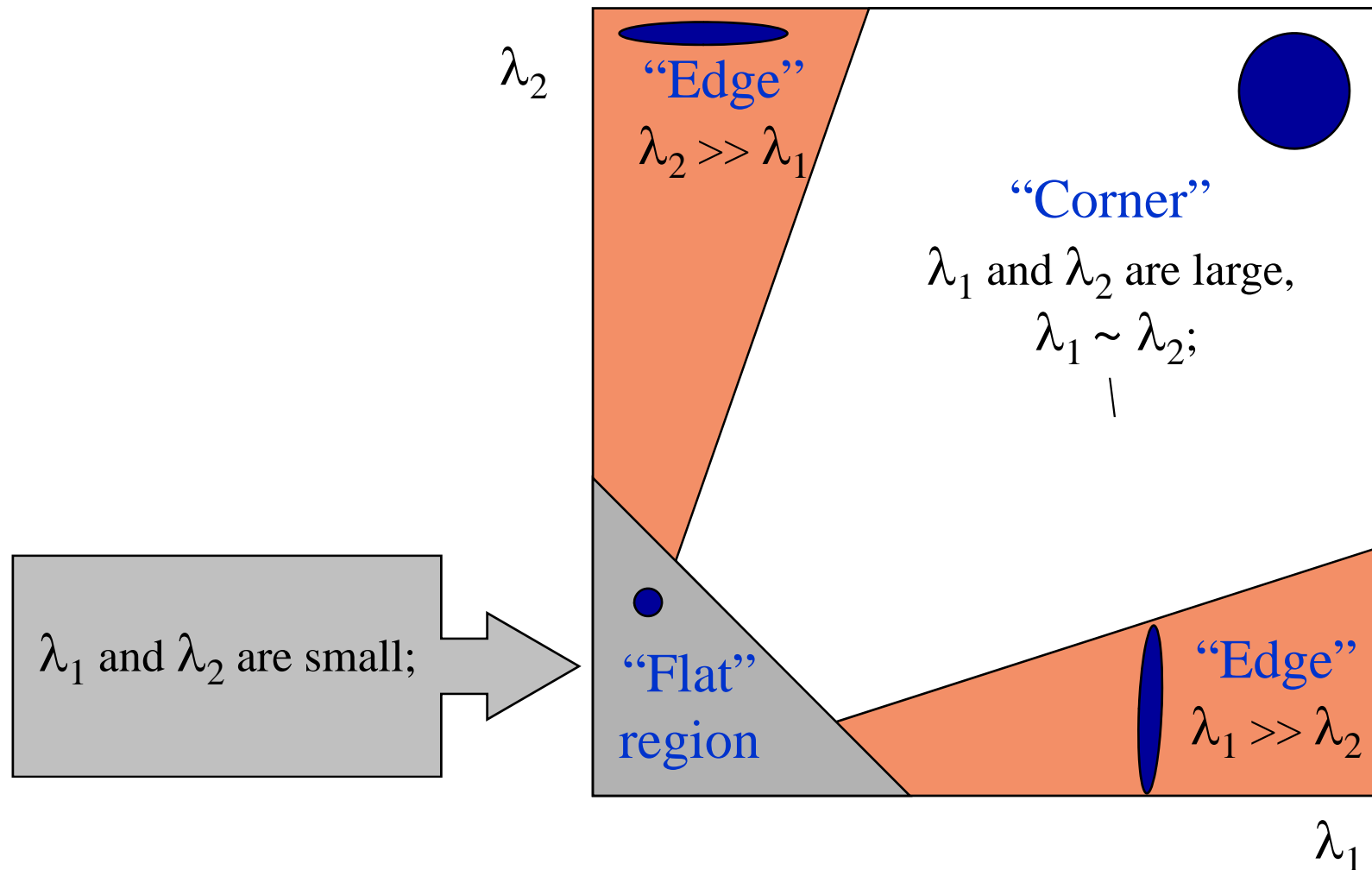
- Auto-correlation matrix

$$A(x, y) = G \otimes \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- captures the structure of the local neighborhood
- measure based on eigenvalues of this matrix
  - 2 strong eigenvalues => interest point
  - 1 strong eigenvalue => contour
  - 0 eigenvalue => uniform region

# Interpreting the eigenvalues

Classification of image points using eigenvalues of autocorrelation matrix:

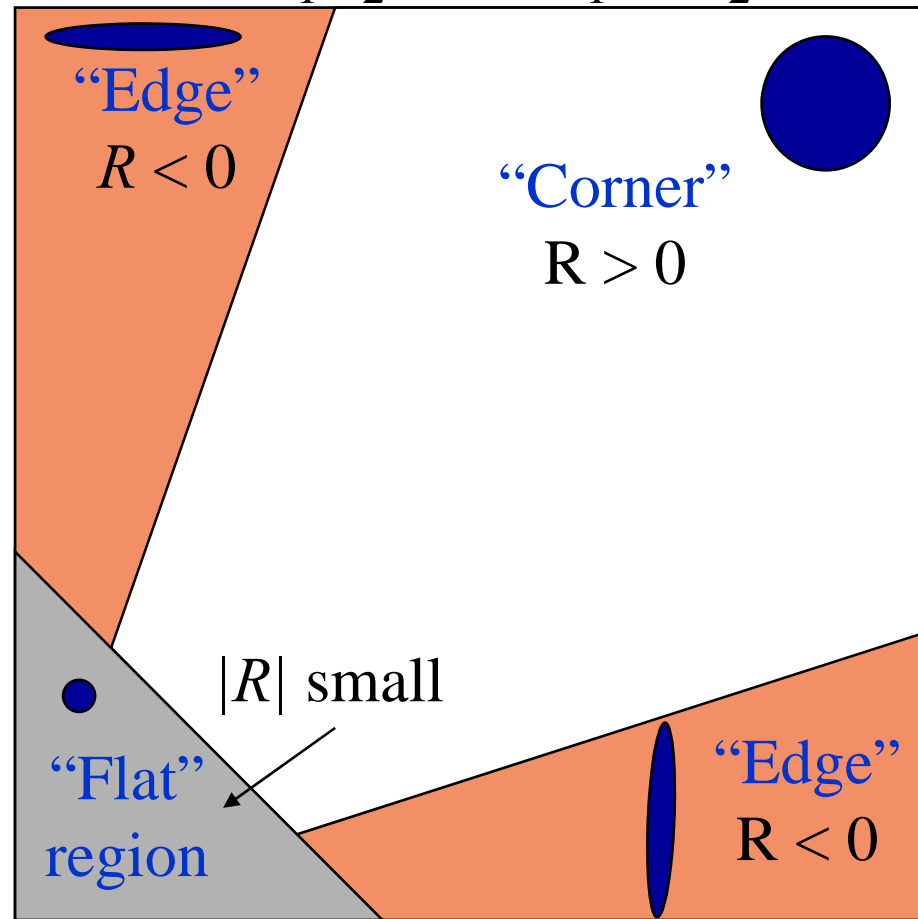


# Corner response function

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$$R = \det(A) - \alpha \text{trace}(A)^2 = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$


$\alpha$ : constant (0.04 to 0.06)



# Harris detector

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- Cornerness function

$$f = \det(A) - k(\text{trace}(A))^2 = \lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2$$


Reduces the effect of a strong contour

- Interest point detection
  - Treshold (absolut, relatif, number of corners)
  - Local maxima

$$f > thresh \wedge \forall x, y \in 8\text{-neighbourhood} \quad f(x, y) \geq f(x', y')$$



# Harris Detector: Steps

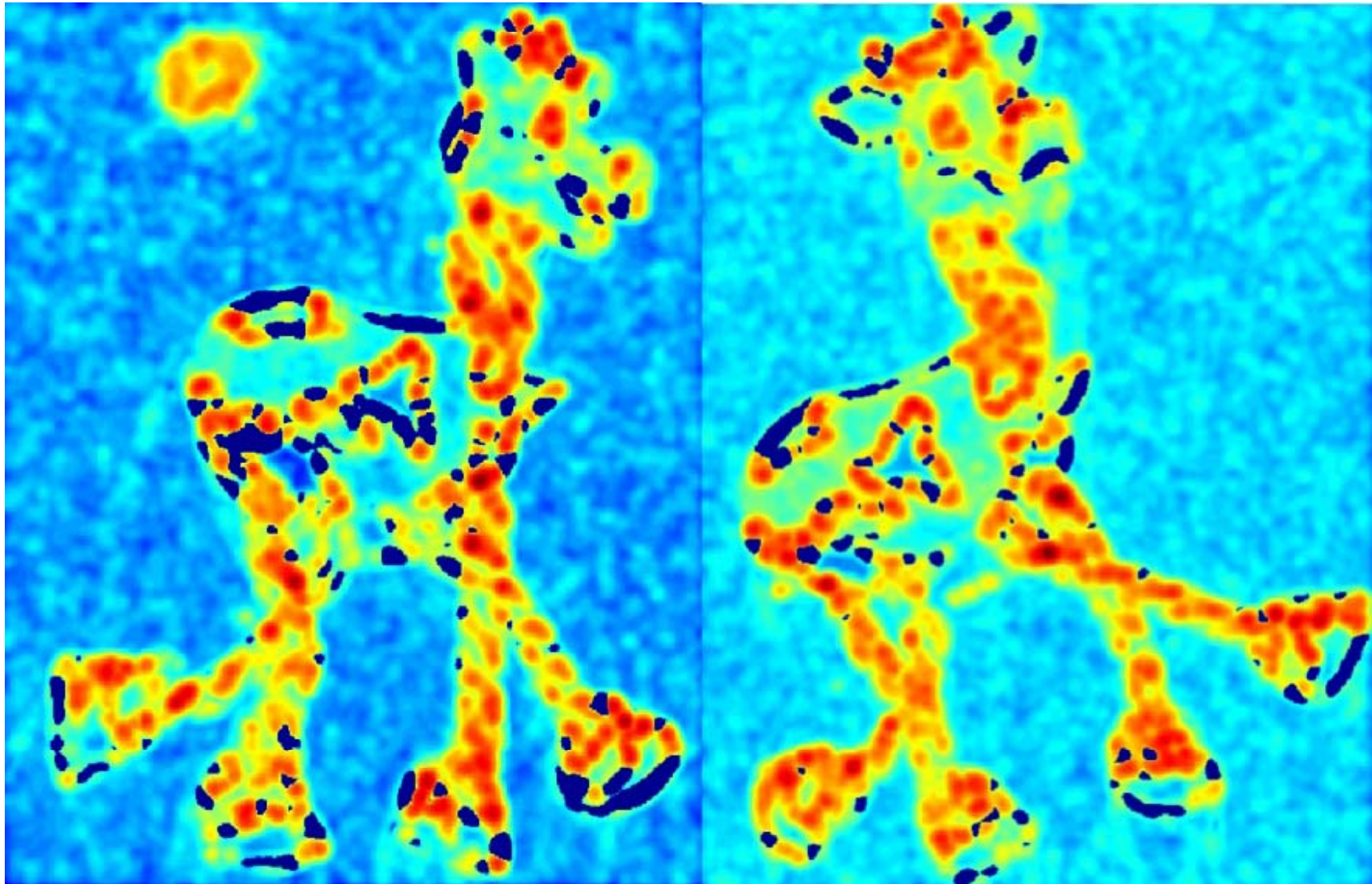
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# Harris Detector: Steps

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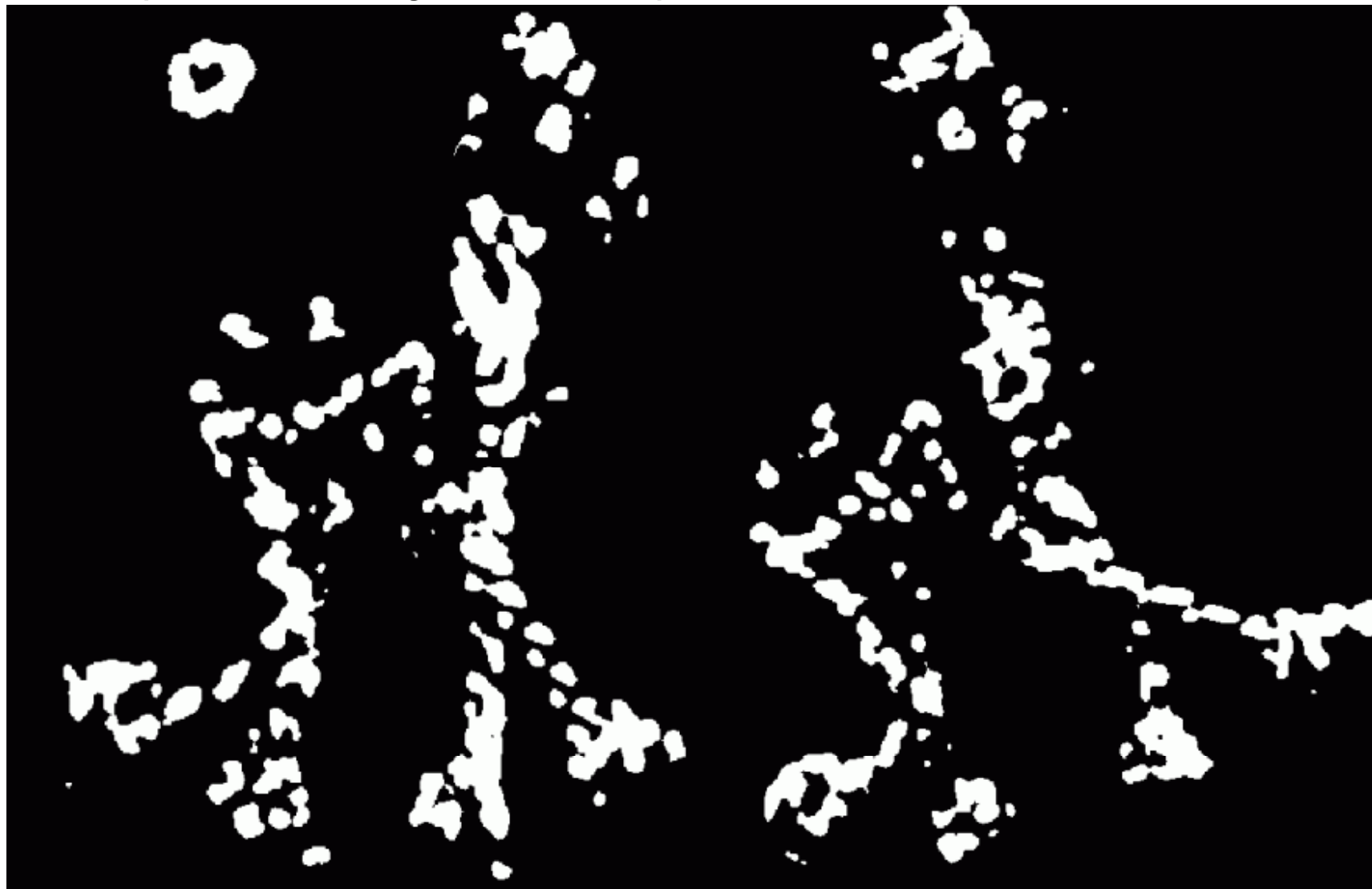
Compute corner response  $R$



# Harris Detector: Steps

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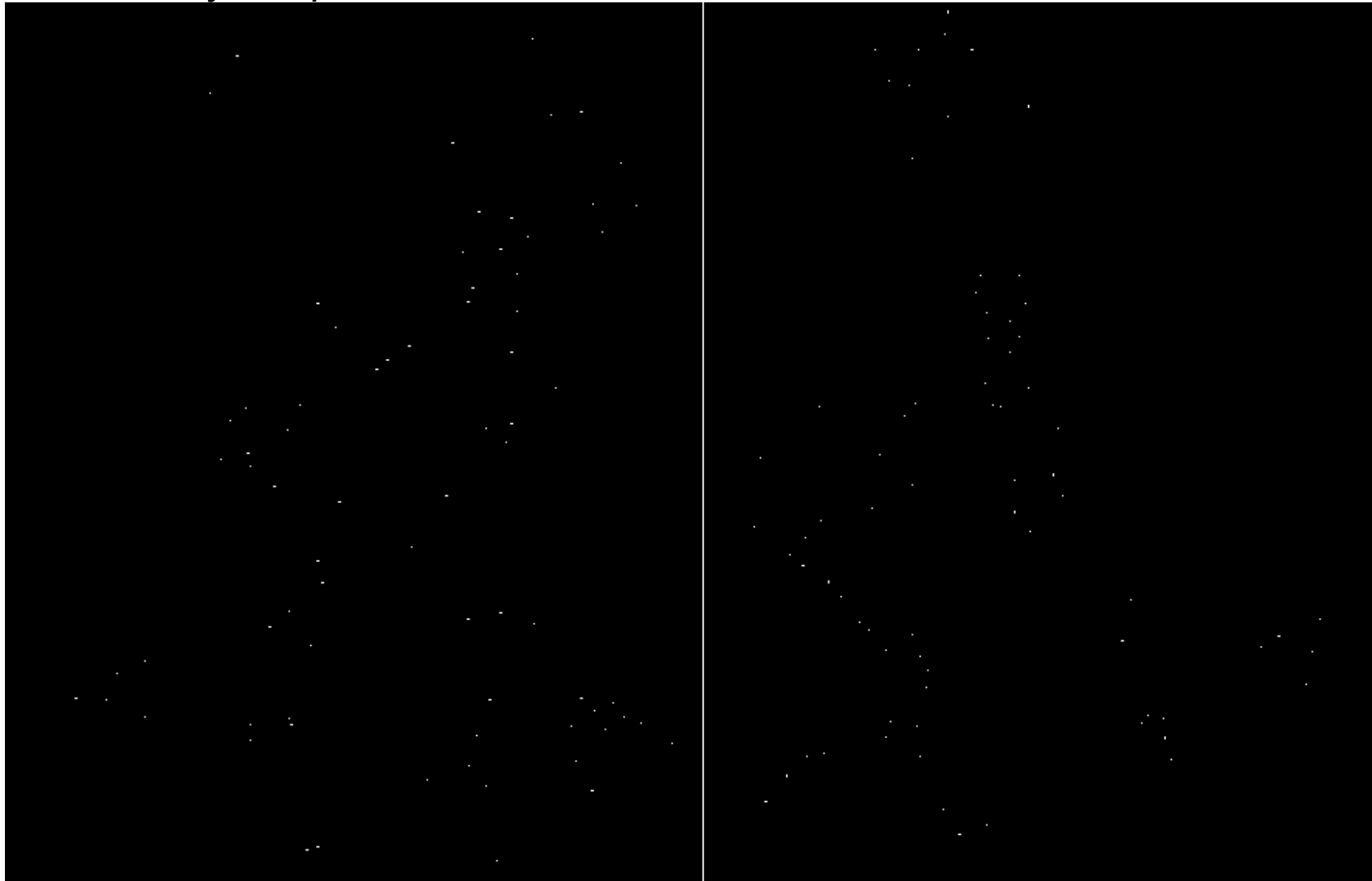
Find points with large corner response:  $R > \text{threshold}$



# Harris Detector: Steps

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Take only the points of local maxima of  $R$





# Harris Detector: Steps

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# Harris detector: Summary of steps

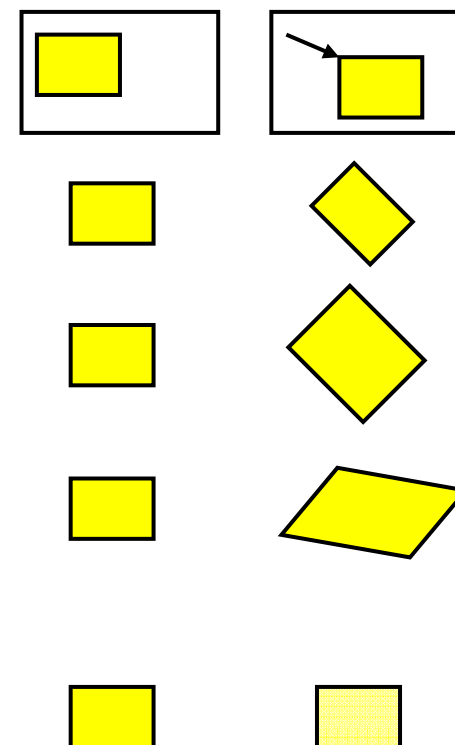
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1. Compute Gaussian derivatives at each pixel
2. Compute second moment matrix  $A$  in a Gaussian window around each pixel
3. Compute corner response function  $R$
4. Threshold  $R$
5. Find local maxima of response function (non-maximum suppression)

# Harris - invariance to transformations

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- Geometric transformations
  - translation
  - rotation
  - similitude (rotation + scale change)
  - affine (valide for local planar objects)
- Photometric transformations
  - Affine intensity changes ( $I \rightarrow a I + b$ )

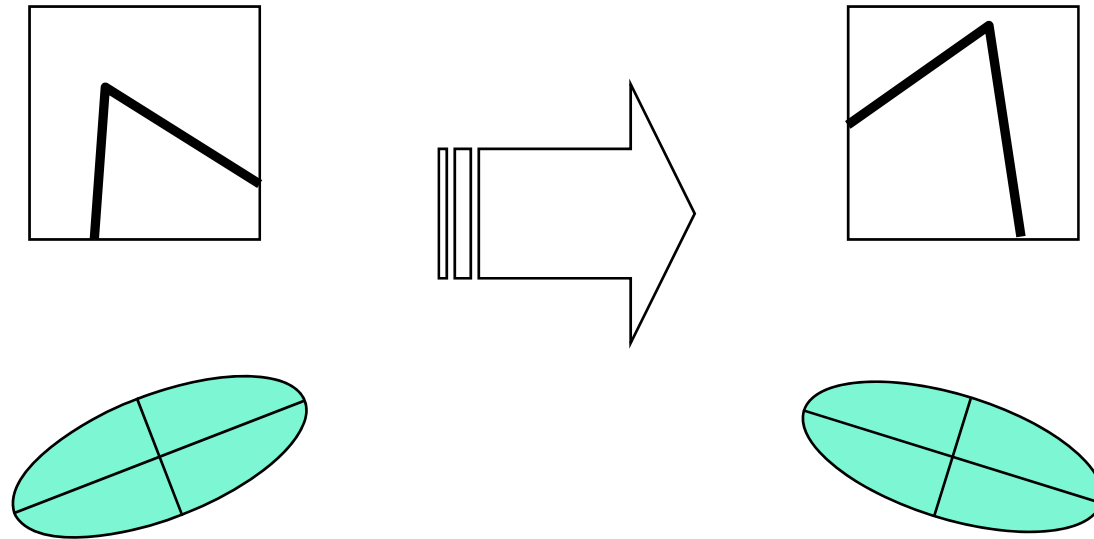




# Harris Detector: Invariance Properties

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- Rotation



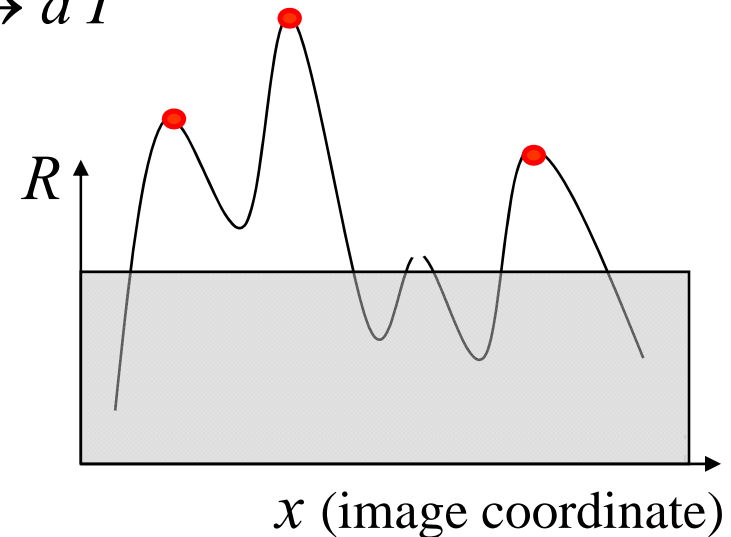
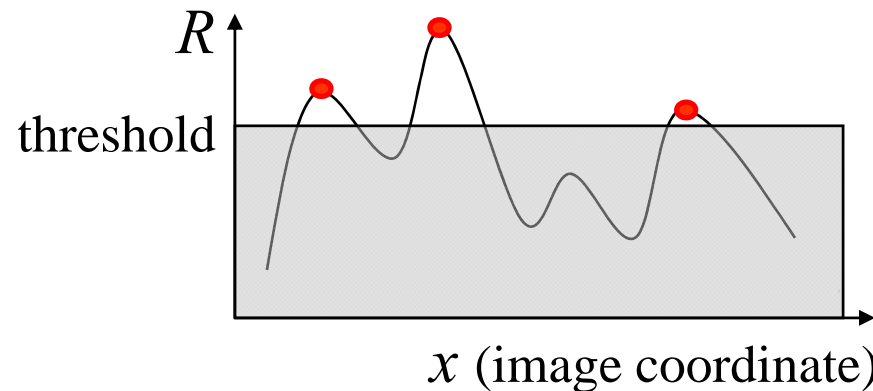
Ellipse rotates but its shape (i.e. eigenvalues)  
remains the same

*Corner response  $R$  is invariant to image rotation*

# Harris Detector: Invariance Properties

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- Affine intensity change
  - ✓ Only derivatives are used => invariance to intensity shift  $I \rightarrow I + b$
  - ✓ Intensity scale:  $I \rightarrow a I$

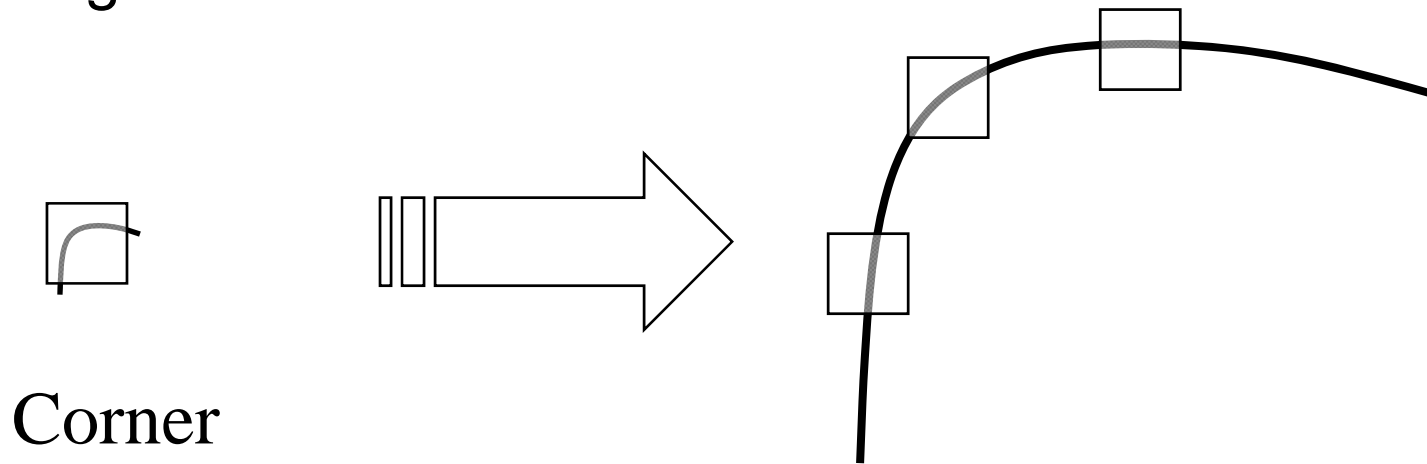


*Partially invariant to affine intensity change,  
dependent on type of threshold*

# Harris Detector: Invariance Properties

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- Scaling



*Not invariant to scaling*

# Comparison of patches - SSD

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Comparison of the intensities in the neighborhood of two interest points

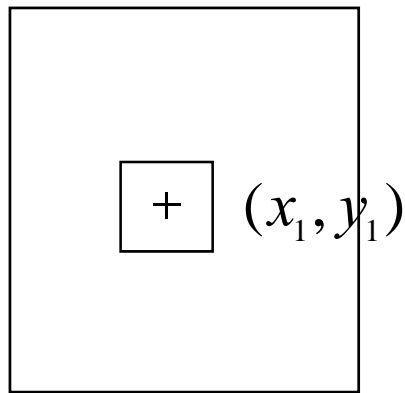


image 1

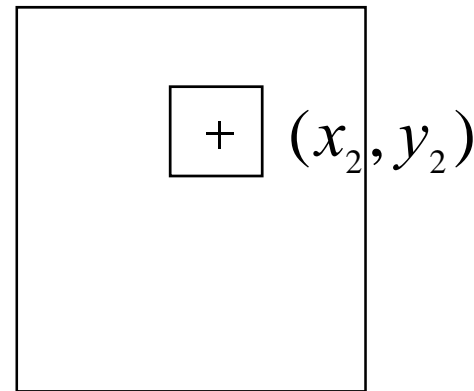


image 2

SSD : sum of square difference

$$\frac{1}{(2N+1)^2} \sum_{i=-N}^N \sum_{j=-N}^N (I_1(x_1 + i, y_1 + j) - I_2(x_2 + i, y_2 + j))^2$$

Small difference values  $\rightarrow$  similar patches

# Comparison of patches

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$$\text{SSD} : \frac{1}{(2N+1)^2} \sum_{i=-N}^N \sum_{j=-N}^N (I_1(x_1 + i, y_1 + j) - I_2(x_2 + i, y_2 + j))^2$$

Invariance to photometric transformations?

Intensity changes ( $I \rightarrow I + b$ )

=> Normalizing with the mean of each patch

$$\frac{1}{(2N+1)^2} \sum_{i=-N}^N \sum_{j=-N}^N ((I_1(x_1 + i, y_1 + j) - m_1) - (I_2(x_2 + i, y_2 + j) - m_2))^2$$

Intensity changes ( $I \rightarrow aI + b$ )

=> Normalizing with the mean and standard deviation of each patch

$$\frac{1}{(2N+1)^2} \sum_{i=-N}^N \sum_{j=-N}^N \left( \frac{I_1(x_1 + i, y_1 + j) - m_1}{\sigma_1} - \frac{I_2(x_2 + i, y_2 + j) - m_2}{\sigma_2} \right)^2$$

# Cross-correlation ZNCC

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zero normalized SSD

$$\frac{1}{(2N+1)^2} \sum_{i=-N}^N \sum_{j=-N}^N \left( \frac{I_1(x_1 + i, y_1 + j) - m_1}{\sigma_1} - \frac{I_2(x_2 + i, y_2 + j) - m_2}{\sigma_2} \right)^2$$



ZNCC: zero normalized cross correlation

$$\frac{1}{(2N+1)^2} \sum_{i=-N}^N \sum_{j=-N}^N \left( \frac{I_1(x_1 + i, y_1 + j) - m_1}{\sigma_1} \right) \cdot \left( \frac{I_2(x_2 + i, y_2 + j) - m_2}{\sigma_2} \right)$$

ZNCC values between -1 and 1, 1 when identical patches  
in practice threshold around 0.5

# Introduction to local descriptors

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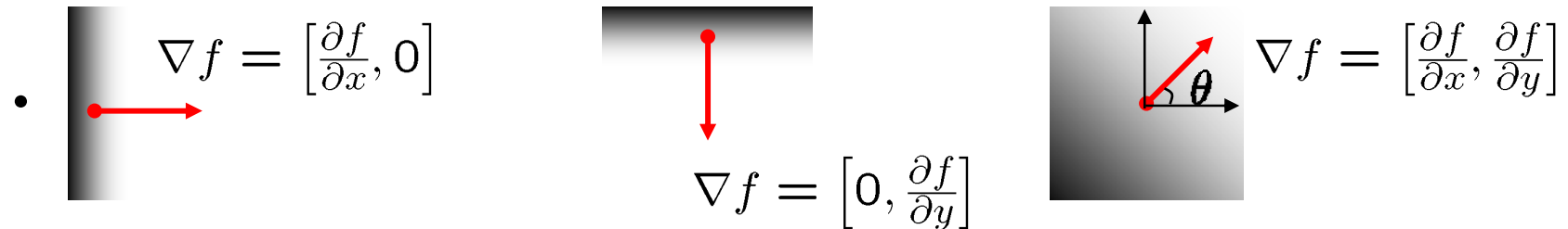
- Greyvalue derivatives
- Differential invariants [Koenderink'87]
- SIFT descriptor [Lowe'99]



# Greyvalue derivatives: Image gradient

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- The gradient of an image:  $\nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$



- The gradient points in the direction of most rapid increase in intensity
- The gradient direction is given by  $\theta = \tan^{-1} \left( \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$ 
  - how does this relate to the direction of the edge?
- The *edge strength* is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

# Differentiation and convolution

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- Recall, for 2D function,  $f(x,y)$ : 
$$\frac{\partial f}{\partial x} = \lim_{\varepsilon \rightarrow 0} \left( \frac{f(x + \varepsilon, y)}{\varepsilon} - \frac{f(x, y)}{\varepsilon} \right)$$

- We could approximate this as 
$$\frac{\partial f}{\partial x} \approx \frac{f(x_{n+1}, y) - f(x_n, y)}{\Delta x}$$

- Convolution with the filter

-1	1
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# Finite difference filters

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- Other approximations of derivative filters exist:

**Prewitt:**  $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$  ;  $M_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$

**Sobel:**  $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$  ;  $M_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

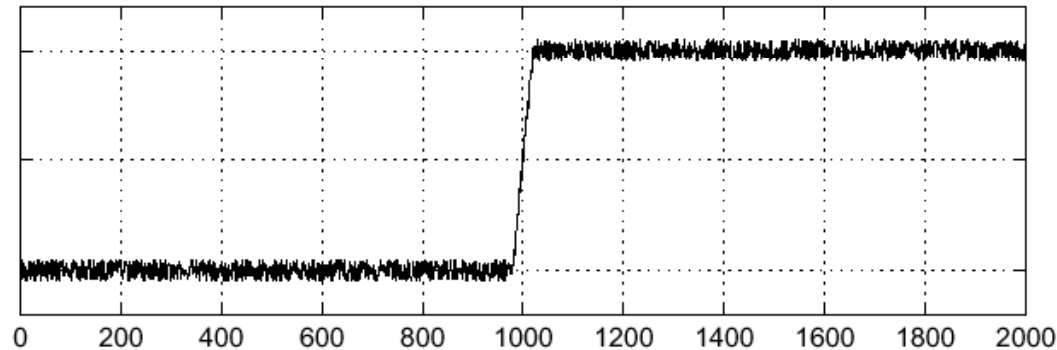
**Roberts:**  $M_x = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$  ;  $M_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

# Effects of noise

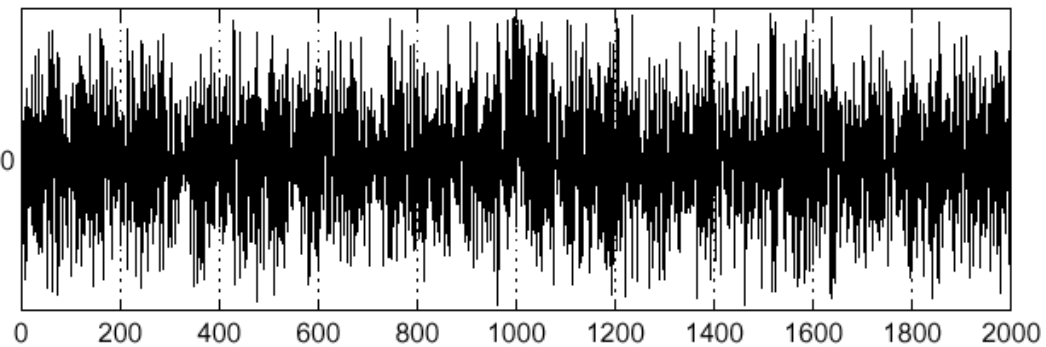
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- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal

$f(x)$

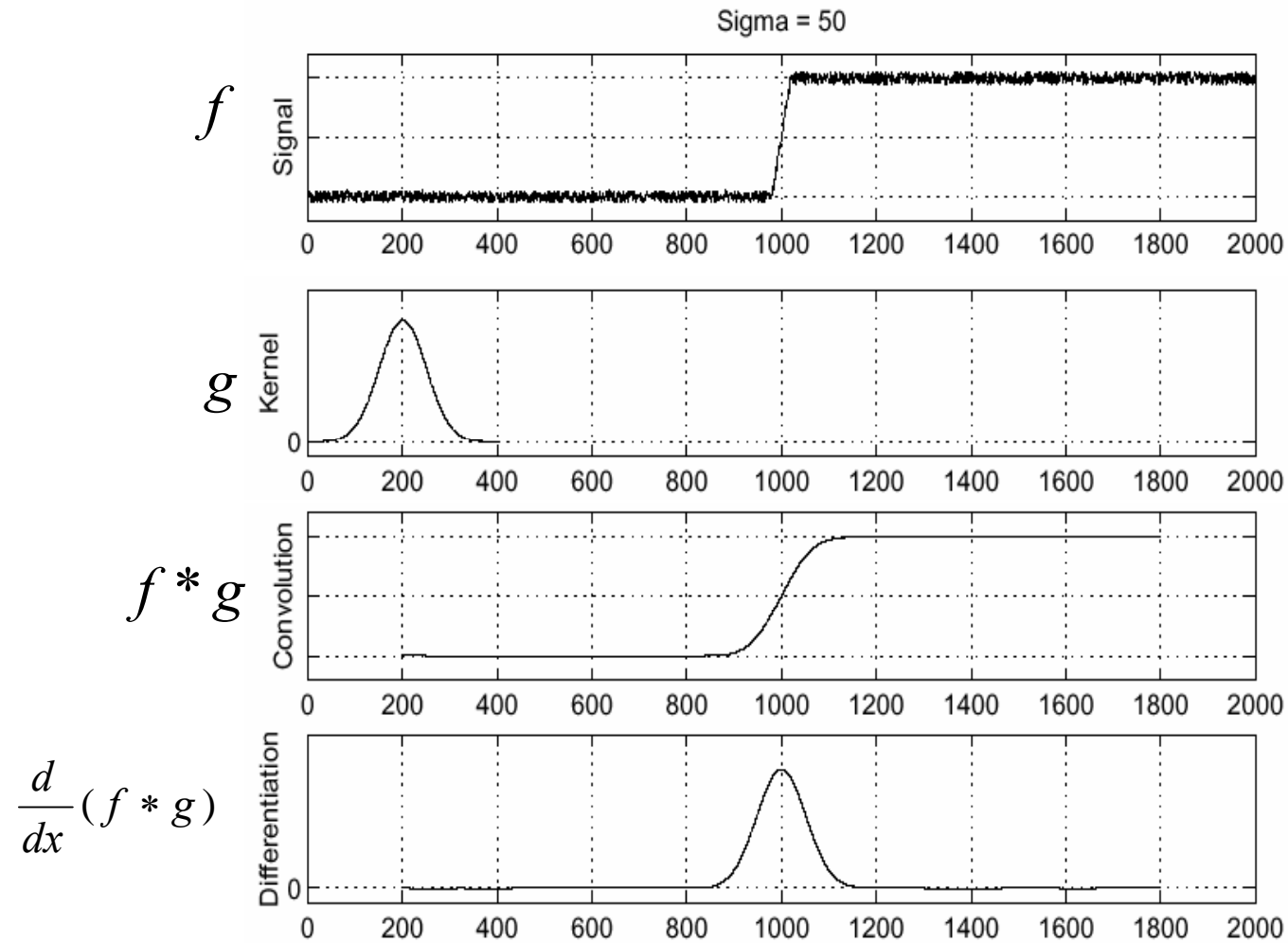


$\frac{d}{dx}f(x)$



- Where is the edge?

# Solution: smooth first

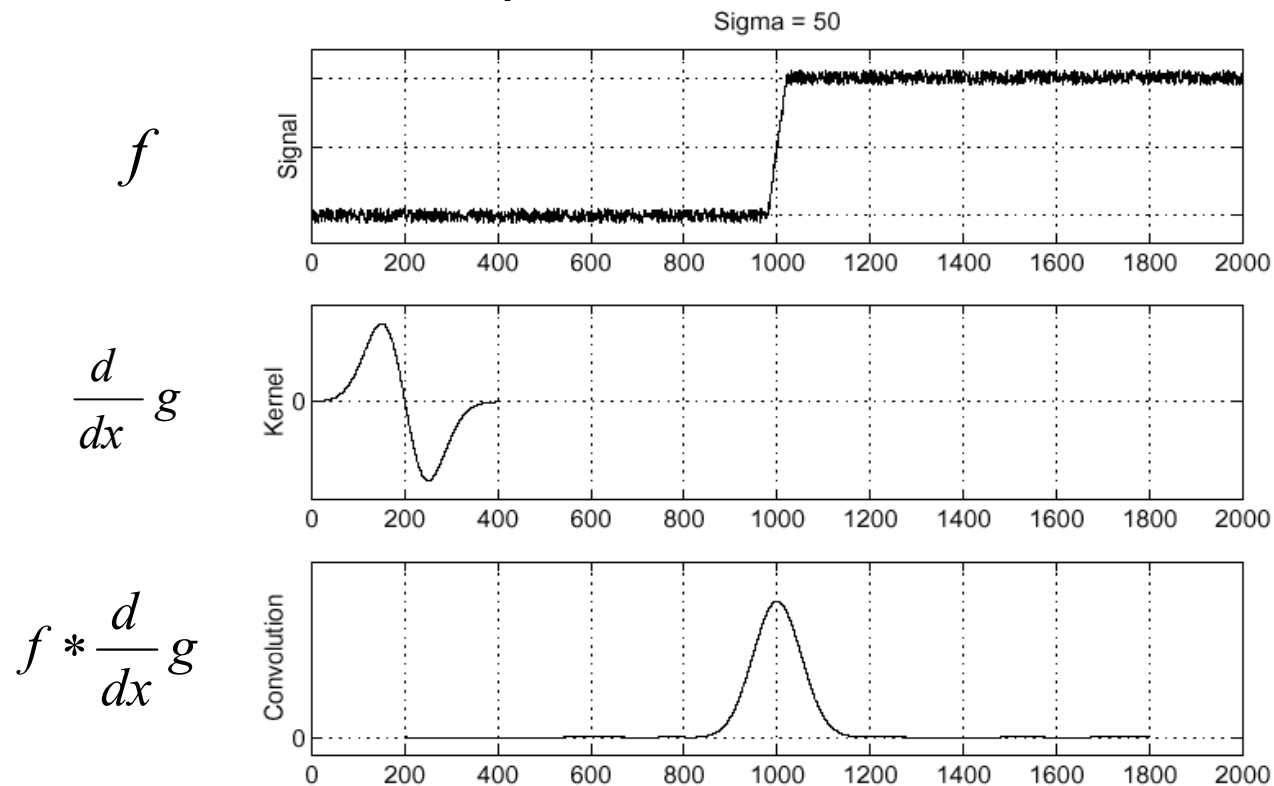


- To find edges, look for peaks in  $\frac{d}{dx}(f * g)$

# Derivative theorem of convolution

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- Differentiation is convolution, and convolution is associative:  $\frac{d}{dx}(f * g) = f * \frac{d}{dx}g$
- This saves us one operation:



# Local descriptors

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- Greyvalue derivatives
  - Convolution with Gaussian derivatives

$$\mathbf{v}(x, y) = \begin{pmatrix} I(x, y) * G(\sigma) \\ I(x, y) * G_x(\sigma) \\ I(x, y) * G_y(\sigma) \\ I(x, y) * G_{xx}(\sigma) \\ I(x, y) * G_{xy}(\sigma) \\ I(x, y) * G_{yy}(\sigma) \\ \vdots \end{pmatrix}$$

$$I(x, y) * G(\sigma) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(x', y', \sigma) I(x - x', y - y') dx' dy'$$

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

# Local descriptors

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Notation for greyvalue derivatives [Koenderink'87]

$$\mathbf{v}(x, y) = \begin{pmatrix} I(x, y) * G(\sigma) \\ I(x, y) * G_x(\sigma) \\ I(x, y) * G_y(\sigma) \\ I(x, y) * G_{xx}(\sigma) \\ I(x, y) * G_{xy}(\sigma) \\ I(x, y) * G_{yy}(\sigma) \\ \vdots \end{pmatrix} = \begin{pmatrix} L(x, y) \\ L_x(x, y) \\ L_y(x, y) \\ L_{xx}(x, y) \\ L_{xy}(x, y) \\ L_{yy}(x, y) \\ \vdots \end{pmatrix}$$

Invariance?



# Local descriptors – rotation invariance

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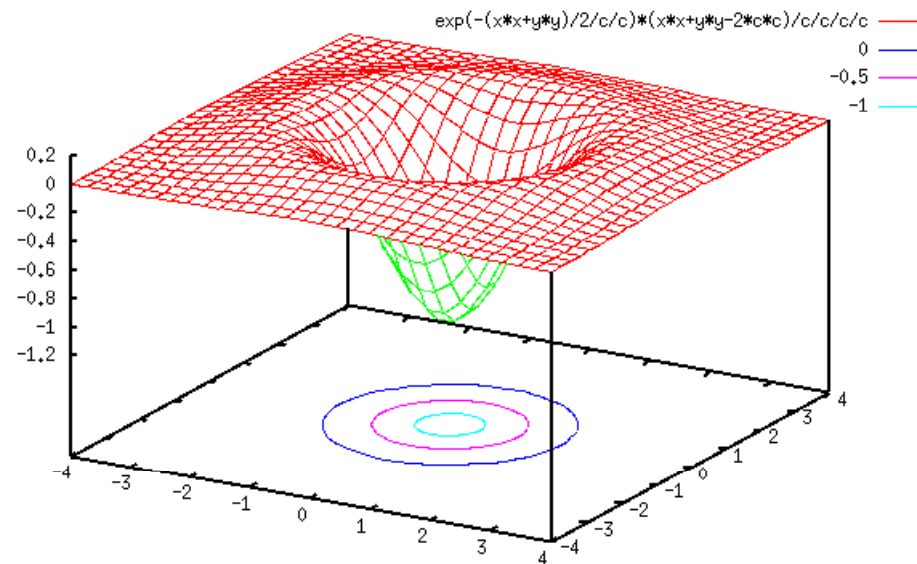
Invariance to image rotation : differential invariants [Koen87]

gradient magnitude	→	$L$
		$L_x L_x + L_y L_y$
		$L_{xx} L_x L_x + 2L_{xy} L_x L_y + L_{yy} L_y L_y$
Laplacian	→	$L_{xx} + L_{yy}$
		$L_{xx} L_{xx} + 2L_{xy} L_{xy} + L_{yy} L_{yy}$
		...
		...
		...
		...

# Laplacian of Gaussian (LOG)

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$$LOG = G_{xx}(\sigma) + G_{yy}(\sigma)$$



# SIFT descriptor [Lowe'99]

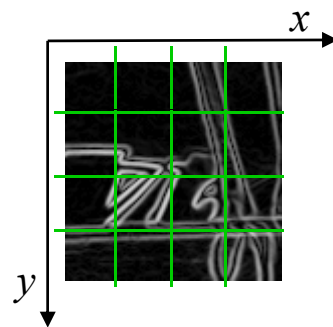
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- Approach
  - 8 orientations of the gradient
  - 4x4 spatial grid
  - Dimension 128
  - soft-assignment to spatial bins
  - normalization of the descriptor to norm one
  - comparison with Euclidean distance

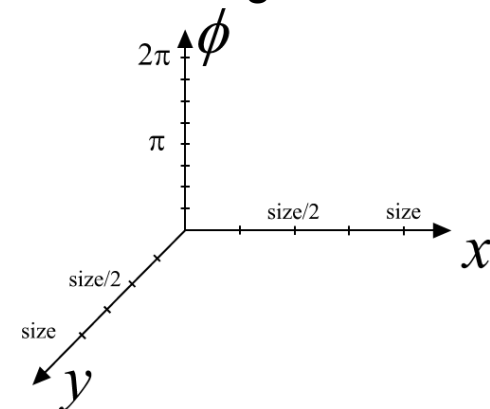
image patch



gradient



3D histogram

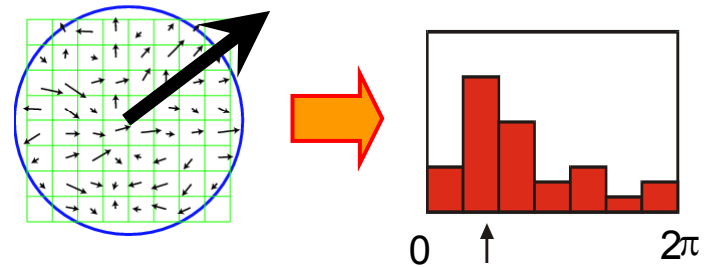


# Local descriptors - rotation invariance

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- Estimation of the dominant orientation

- extract gradient orientation
- histogram over gradient orientation
- peak in this histogram



- Rotate patch in dominant direction



# Local descriptors – illumination change

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- Robustness to illumination changes

in case of an affine transformation  $I_1(\mathbf{x}) = aI_2(\mathbf{x}) + b$

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# Local descriptors – illumination change

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- Normalization of the image patch with mean and variance

# Invariance to scale changes

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- Scale change between two images
- Scale factor  $s$  can be eliminated
- Support region for calculation!!
  - In case of a convolution with Gaussian derivatives defined by  $\sigma$

$$I(x, y) * G(\sigma) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(x', y', \sigma) I(x - x', y - y') dx' dy'$$

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$