Optical flow

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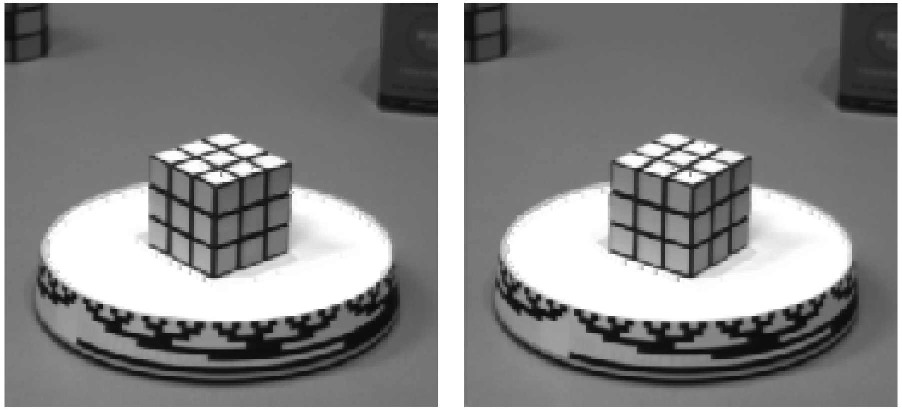
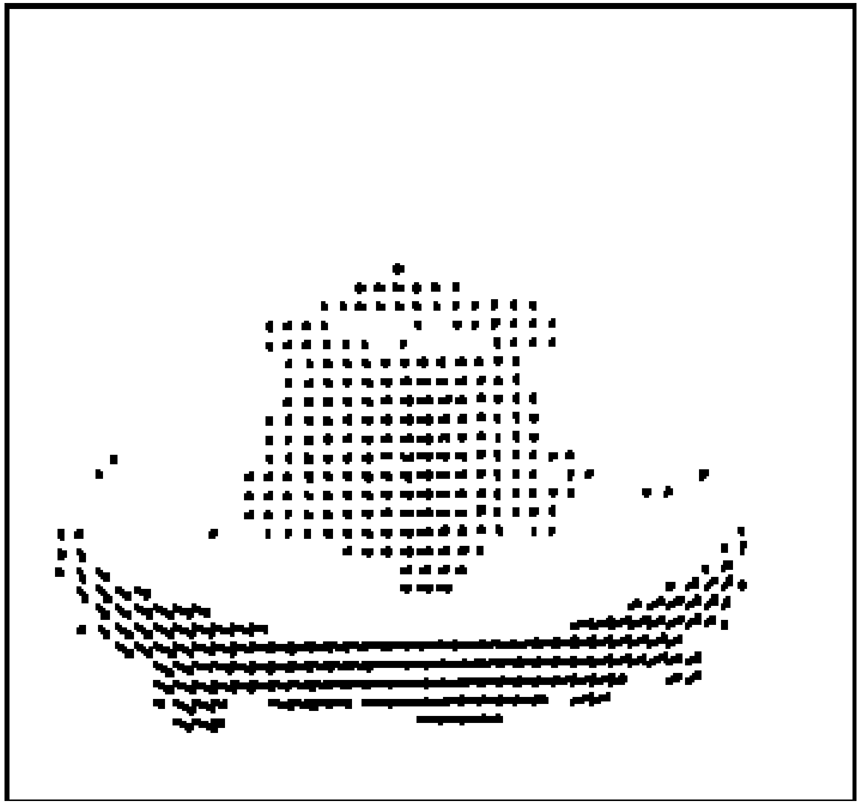
Motion field

- The motion field is the projection of the 3D scene motion into the image

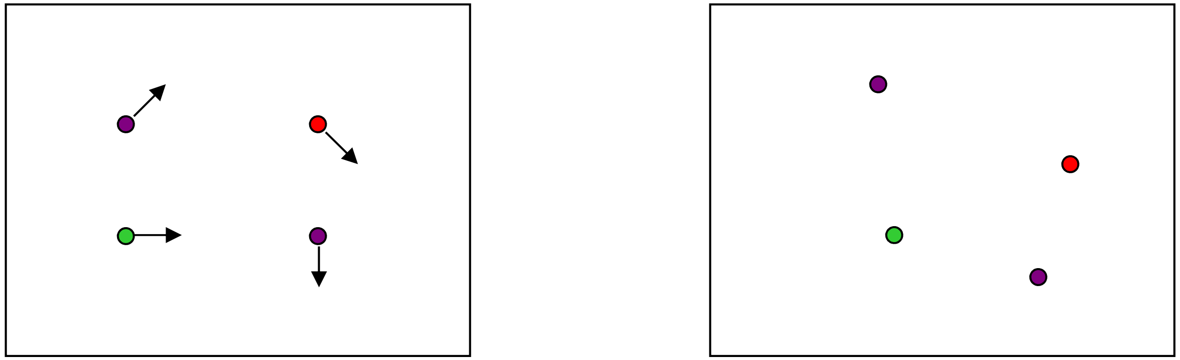




Optical flow

- Definition: optical flow is the *apparent* motion of brightness patterns in the image
- Ideally, optical flow would be the same as the motion field
- Have to be careful: apparent motion can be caused by lighting changes without any actual motion
  - Think of a uniform rotating sphere under fixed lighting vs. a stationary sphere under moving illumination

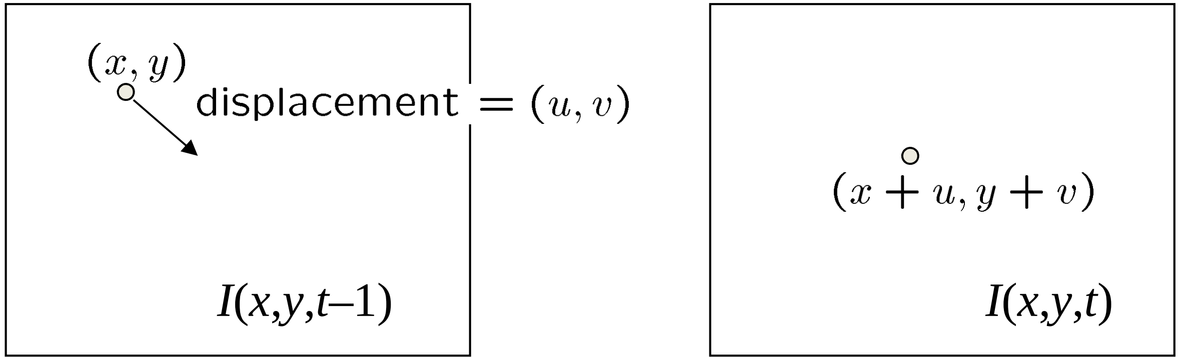
Estimating optical flow



$I(x,y,t–1)$ $I(x,y,t)$

- Given two subsequent frames, estimate the apparent motion field *u*(*x*,*y*) and *v*(*x*,*y*) between them
- Key assumptions
  - Brightness constancy: projection of the same point looks the same in every frame
  - Small motion: points do not move very far
  - Spatial coherence: points move like their neighbors

The brightness constancy constraint



Brightness Constancy Equation:

$$I(x, y, t-1) = I(x + u(x, y), y + v(x, y), t)$$

Linearizing the right side using Taylor expansion:

$$I(x, y, t-1) \approx I(x, y, t) + I_x u(x, y) + I_y v(x, y)$$

Hence, $I_x u + I_y v + I_t \approx 0$

The brightness constancy constraint

$$I_x u + I_y v + I_t = 0$$

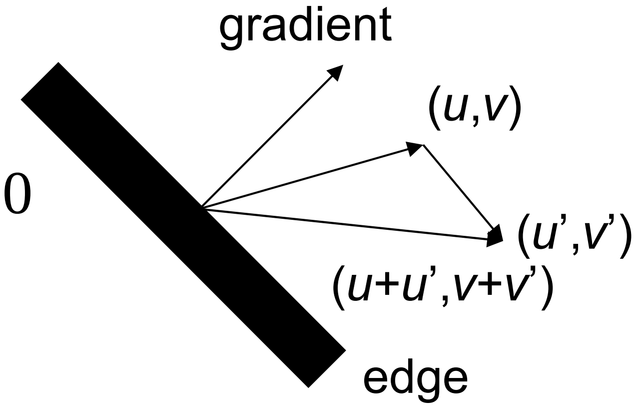
- How many equations and unknowns per pixel?
  - One equation, two unknowns

- What does this constraint mean?

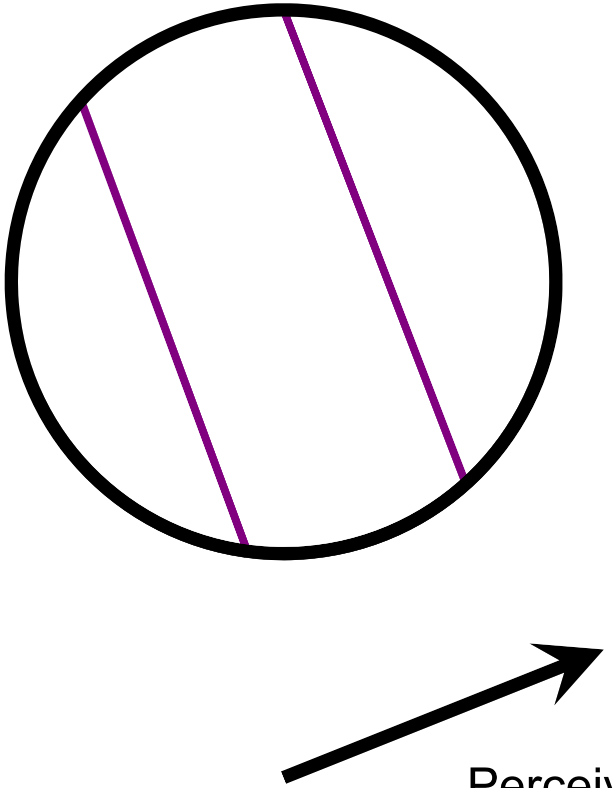
$$\nabla I \cdot (u,v) + I_t = 0$$

- The component of the flow perpendicular to the gradient (i.e., parallel to the edge) is unknown

If (u, v) satisfies the equation,
so does $(u+u', v+v')$ if $\nabla I \cdot (u',v') = 0$

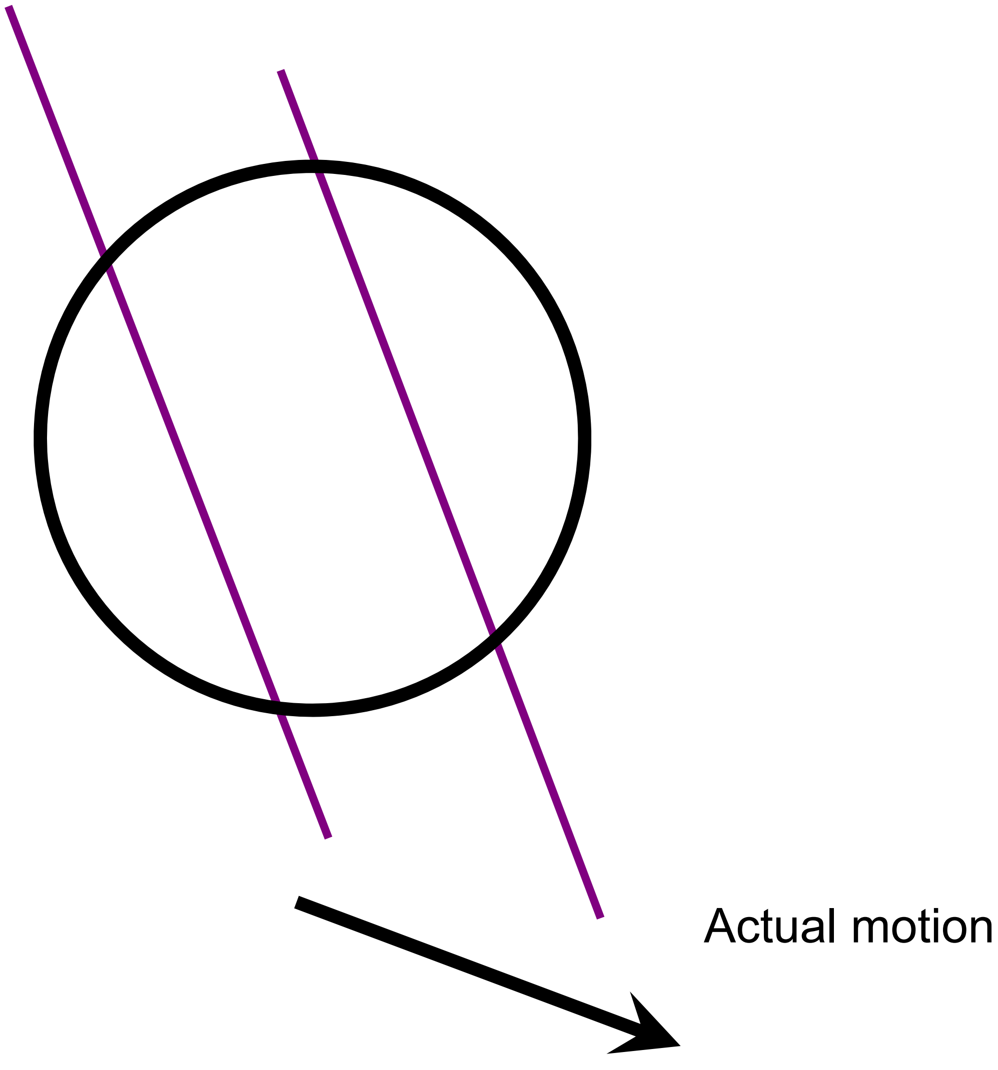


The aperture problem



Perceived motion

The aperture problem



Solving the aperture problem

- How to get more equations for a pixel?
- **Spatial coherence constraint:** pretend the pixel's neighbors have the same (u,v)
  - E.g., if we use a 5x5 window, that gives us 25 equations per pixel

$$\begin{bmatrix} I_x(\mathbf{x}_1) & I_y(\mathbf{x}_1) \\ I_x(\mathbf{x}_2) & I_y(\mathbf{x}_2) \\ \vdots & \vdots \\ I_x(\mathbf{x}_n) & I_y(\mathbf{x}_n) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} I_t(\mathbf{x}_1) \\ I_t(\mathbf{x}_2) \\ \vdots \\ I_t(\mathbf{x}_n) \end{bmatrix}$$

B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *International Joint Conference on Artificial Intelligence*,1981.

Lucas-Kanade flow

- Linear least squares problem

$$\begin{bmatrix} I_x(\mathbf{x}_1) & I_y(\mathbf{x}_1) \\ I_x(\mathbf{x}_2) & I_y(\mathbf{x}_2) \\ \vdots & \vdots \\ I_x(\mathbf{x}_n) & I_y(\mathbf{x}_n) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} I_t(\mathbf{x}_1) \\ I_t(\mathbf{x}_2) \\ \vdots \\ I_t(\mathbf{x}_n) \end{bmatrix}$$

$$\underset{n\times 2}{\mathbf{A}} \; \underset{2\times 1}{\mathbf{d}} = \underset{n\times 1}{\mathbf{b}}$$

Solution given by $(\mathbf{A}^T\mathbf{A})\mathbf{d} = \mathbf{A}^T\mathbf{b}$

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

The summations are over all pixels in the window

Lucas-Kanade flow

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

- Recall the Harris corner detector: M = A^TA is the *second moment matrix*
- When is the system solvable?
  - By looking at the eigenvalues of the second moment matrix
  - The eigenvectors and eigenvalues of M relate to edge direction and magnitude
  - The eigenvector associated with the larger eigenvalue points in the direction of fastest intensity change, and the other eigenvector is orthogonal to it

Uniform region

- – gradients have small magnitude
- – small λ_1, small λ_2
- – system is ill-conditioned

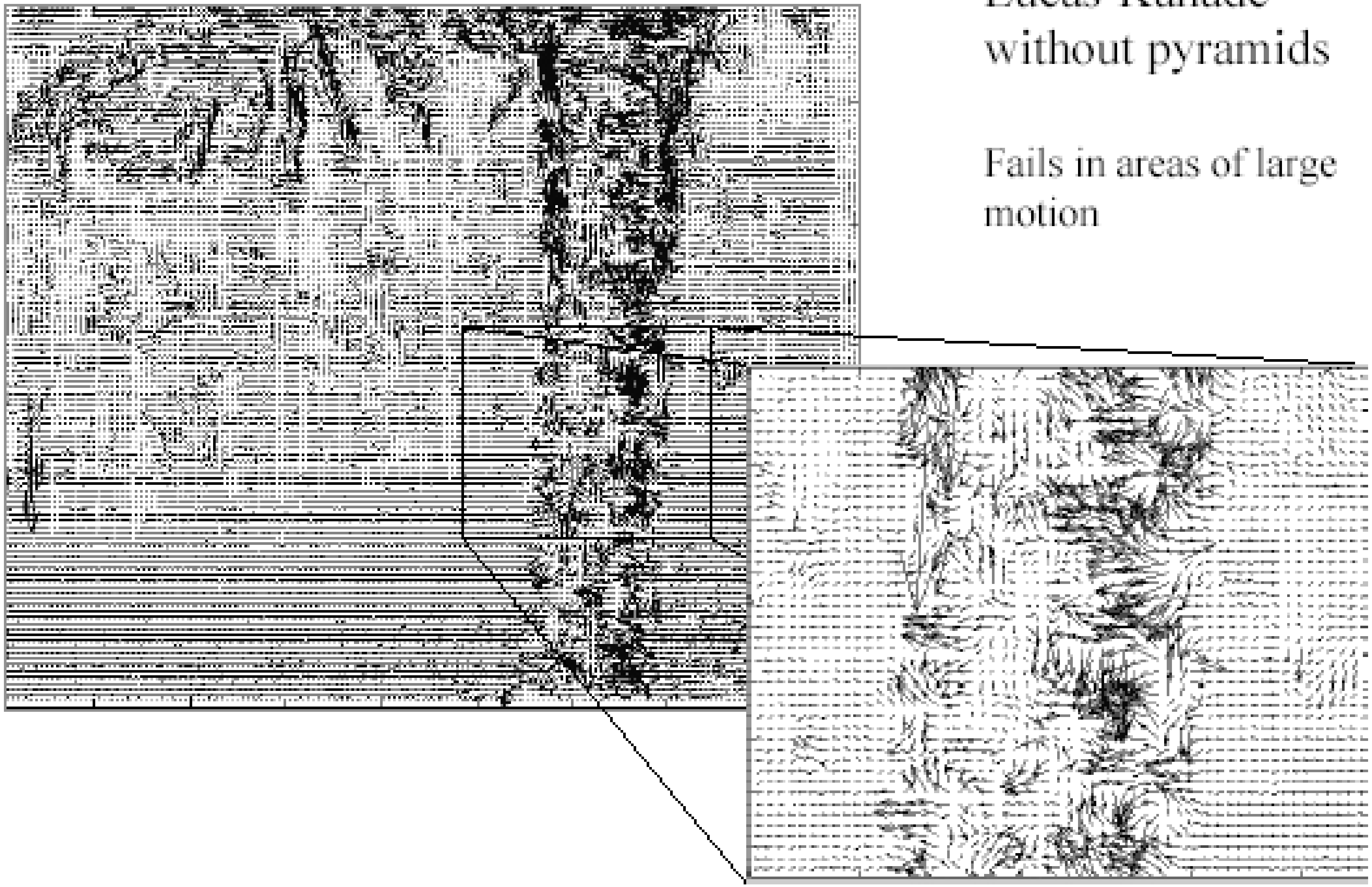
Edge

- gradients have one dominant direction
- large λ_1, small λ_2
- system is ill-conditioned

High-texture or corner region

- gradients have different directions, large magnitudes
- large λ_1, large λ_2
- system is well-conditioned

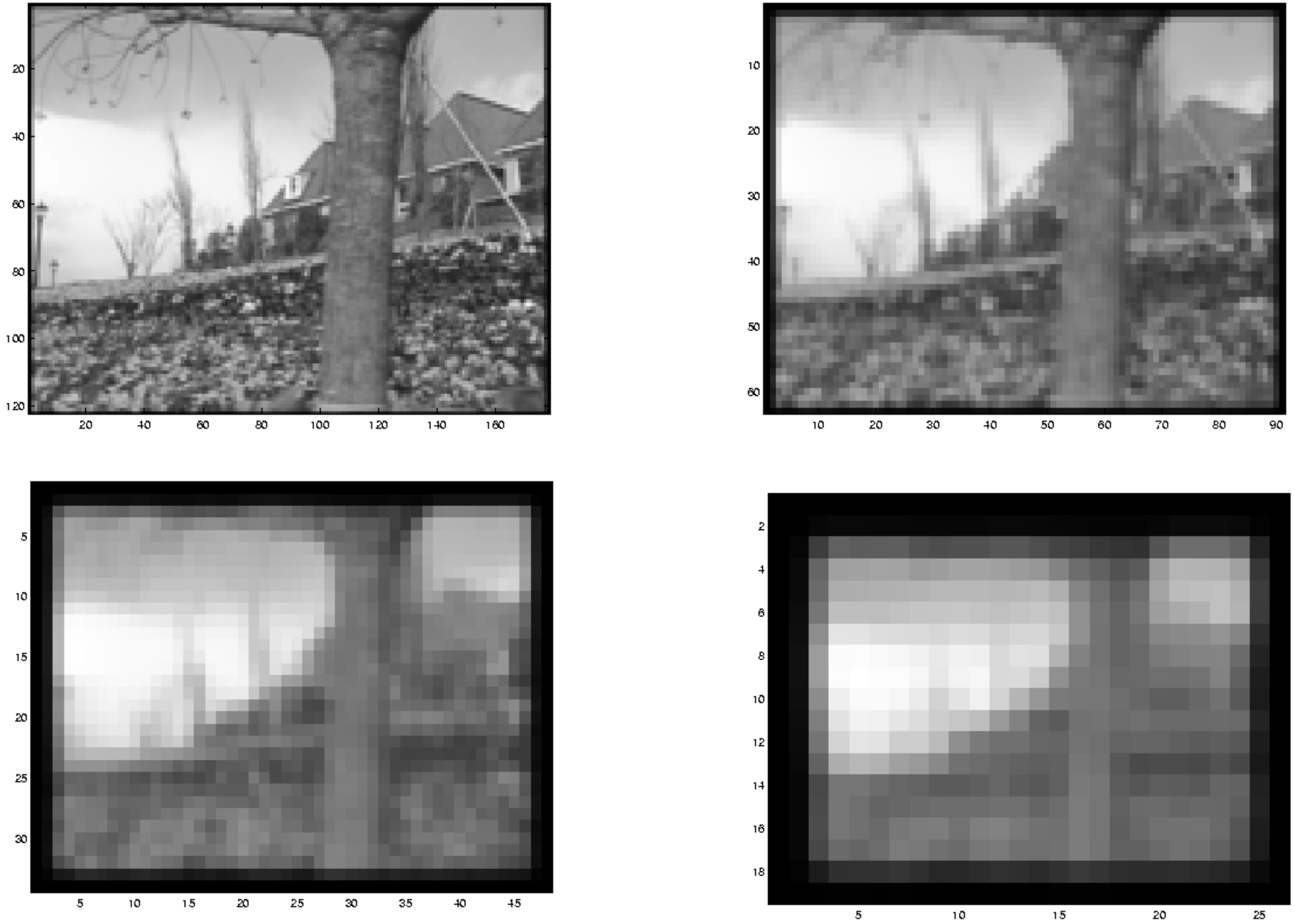
Optical Flow Results



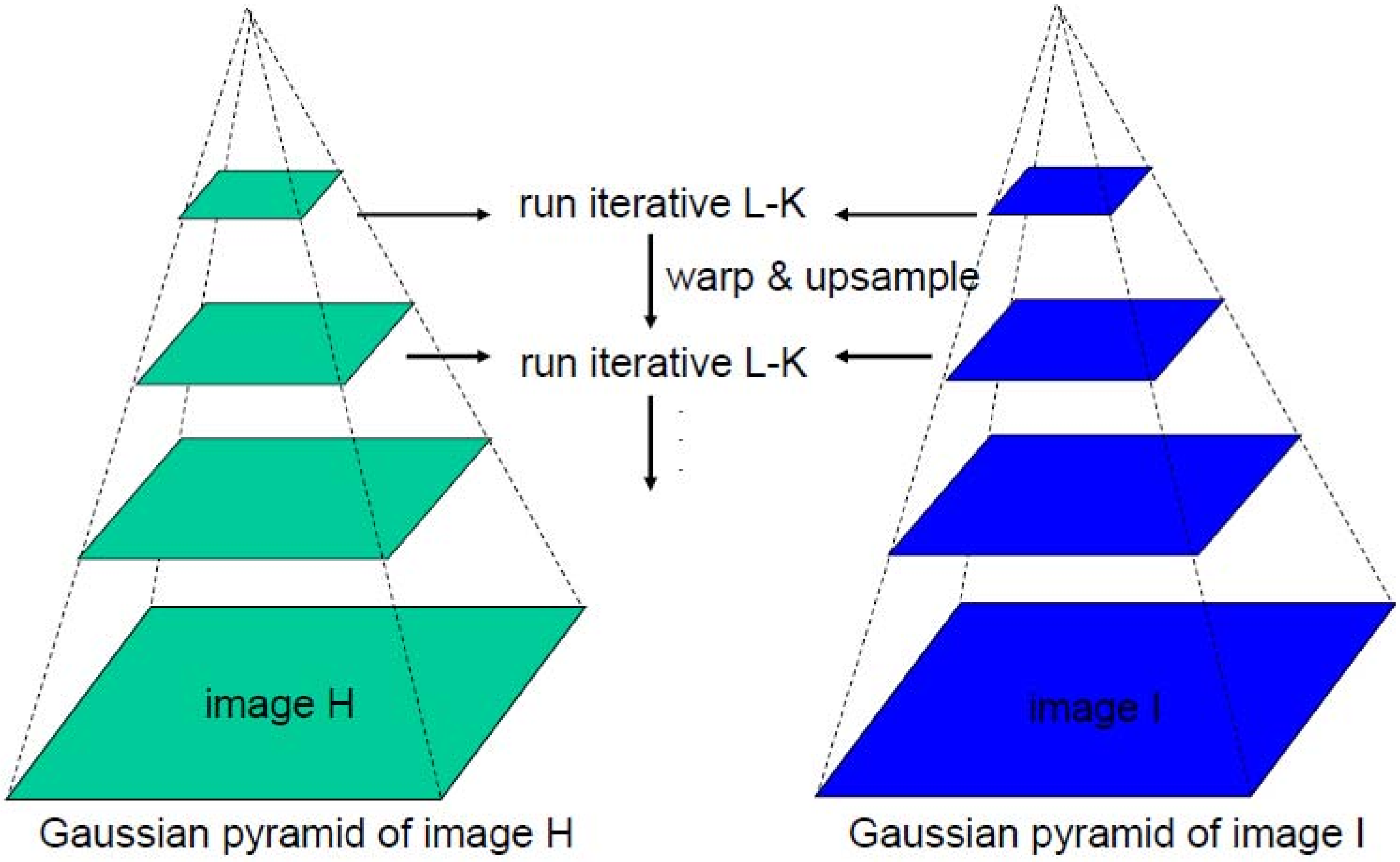
Lucas-Kanade
without pyramids

Fails in areas of large
motion

Multi-resolution registration



Coarse to fine optical flow estimation



run iterative L-K

warp & upsample

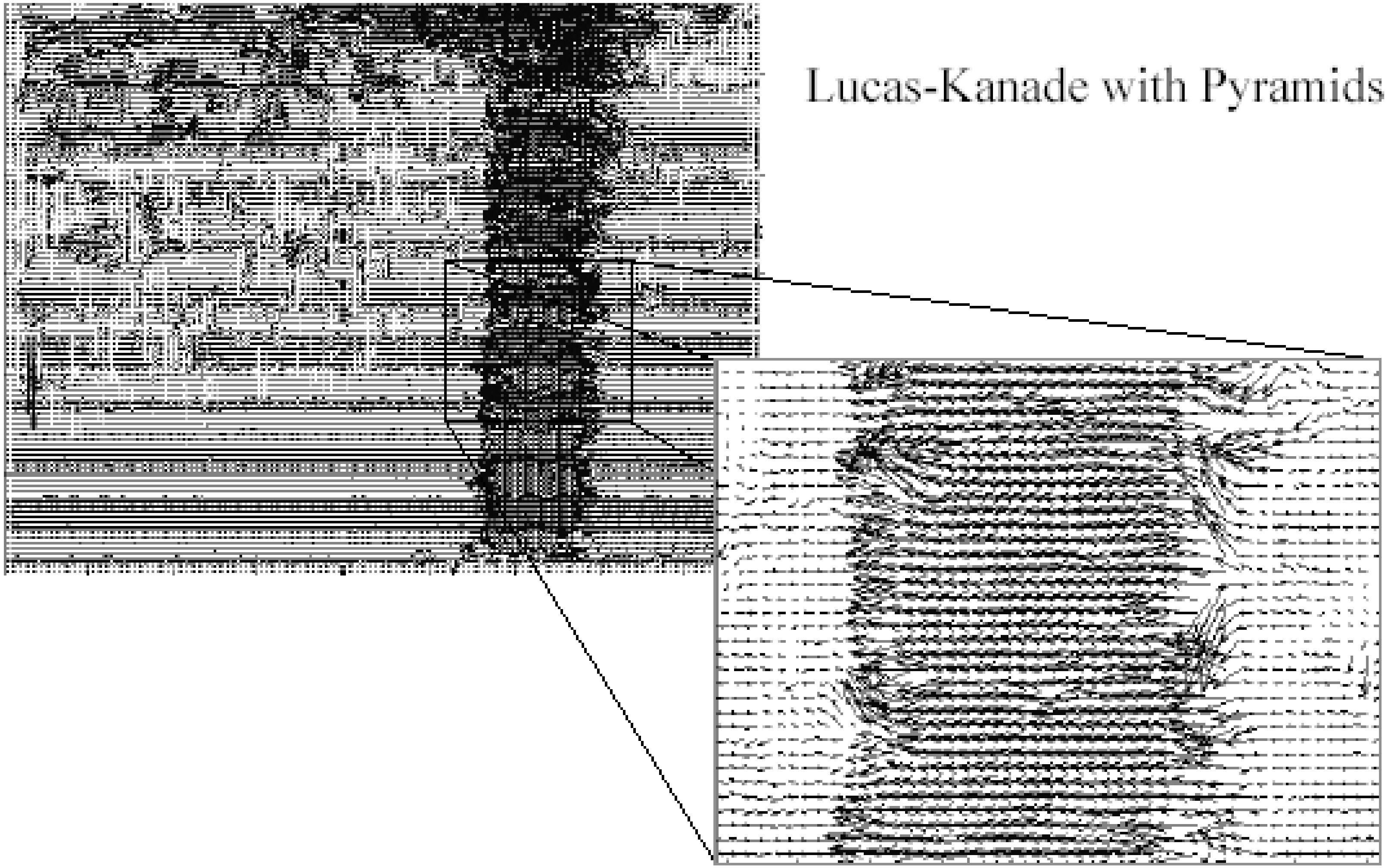
run iterative L-K

.
.
.

Gaussian pyramid of image H

Gaussian pyramid of image I

Optical Flow Results



Lucas-Kanade with Pyramids

Horn & Schunck algorithm

Additional smoothness constraint :

- nearby point have similar optical flow
- Addition constraint $||\nabla u||^2, \; ||\nabla v||^2$

$$e_s = \iint ((u_x^2 + u_y^2) + (v_x^2 + v_y^2))dxdy,$$

B.K.P. Horn and B.G. Schunck, "Determining optical flow." *Artificial Intelligence*,1981

Horn & Schunck algorithm

Additional smoothness constraint :

$$e_s = \iint ((u_x^2 + u_y^2) + (v_x^2 + v_y^2))dxdy,$$

besides OF constraint equation term

$$e_c = \iint (I_x u + I_y v + I_t)^2 dxdy,$$

minimize $e_s + \lambda e_c$

λ regularization parameter

B.K.P. Horn and B.G. Schunck, "Determining optical flow." *Artificial Intelligence*,1981

Horn & Schunck algorithm

$$E(u(x,y),v(x,y)) = \iint \underbrace{(I_x u + I_y v + I_t)^2}_{\substack{\text{Data term}\\ \text{brightness}\\ \text{constancy}}} + \alpha \underbrace{((u_x^2 + u_y^2) + (v_x^2 + v_y^2))}_{\substack{\text{Smoothness}\\ \text{term}}} dxdy$$

$$E(u,v) = \int_{\Omega} F(x,y,u,v,u_x,u_y,v_x,v_y)dxdy$$

Euler-Lagrange equations

$$F_u - \frac{\partial}{\partial x} F_{u_x} - \frac{\partial}{\partial y} F_{u_y} = 0 \qquad F_v - \frac{\partial}{\partial x} F_{v_x} - \frac{\partial}{\partial y} F_{v_y} = 0$$

According to the calculus of variations, a minimizer of E must fulfill the Euler-Lagrange equations

Horn & Schunck

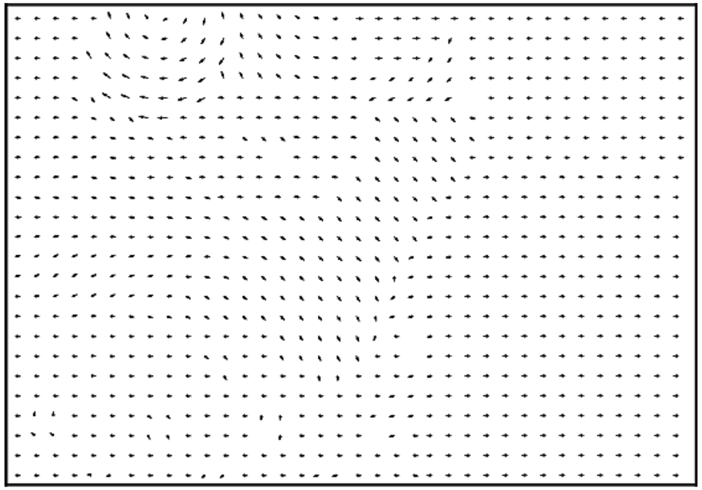
Solution :

1. Coupled PDEs solved using iterative methods and finite differences

2. Information spreads from corner-type patterns

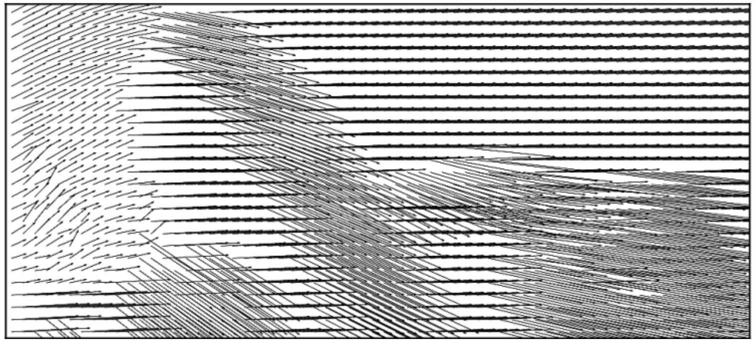
Horn & Schunck

- Works well for small displacements
  - For example Middlebury sequence



Large displacement estimation in optical flow

- Large displacement is still an open problem in optical flow estimation



MPI Sintel dataset

Large displacement optical flow

- Classical optical flow [Horn and Schunck 1981]
  - energy: $E(\mathbf{w}) = \iint \underbrace{E_{data}}_{\text{color/gradient constancy}} + \alpha \underbrace{E_{smooth}}_{\text{smoothness constraint}} \mathbf{dx}$
  - minimization using a coarse-to-fine scheme

- Large displacement approaches:
  - LDOF [Brox and Malik 2011]
    a matching term, penalizing the difference between flow and HOG matches

    $$E(\mathbf{w}) = \iint E_{data} + \alpha E_{smooth} + \beta E_{match} \mathbf{dx}$$

  - MDP-Flow2 [Xu *et al.* 2012]
    expensive fusion of matches (SIFT + PatchMatch) and estimated flow at each level

  - DeepFlow [Weinzaepfel *et al.* 2013]
    deep matching + flow refinement with variational approach