Sparse Learned Representations for Image Restoration

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What this talk is about

- The learning of compact representations of images adapted to restoration tasks.
- A multiscale method to learn such representations.
- Various formulations for image and video processing.

1. Sparse representations for image denoising
2. Multiscale extension
3. Various formulations for image and video processing

1. Sparse representations for image denoising
2. Multiscale extension
3. Various formulations for image and video processing

Sparse representations for image restoration

$$\underbrace{\mathbf{y}}_{\text{measurements}} = \underbrace{\mathbf{x}_{orig}}_{\text{original image}} + \underbrace{\mathbf{w}}_{\text{white Gaussian noise}}$$

Energy minimization problem - MAP estimation:

$$E(\mathbf{x}) = \underbrace{||\mathbf{y} - \mathbf{x}||_2^2}_{\text{relation to measurements}} + \underbrace{Pr(\mathbf{x})}_{prior}$$

Sparse representations for image restoration

Some classical priors

- Smoothness $\lambda||\mathbf{Lx}||_2^2$
- Total variation $\lambda||\nabla\mathbf{x}||_2^2$
- Wavelet sparsity $\lambda||\mathbf{Wx}||_1$
- ...

Sparse representations for image restoration

Sparsity and redundancy

$Pr(\mathbf{x}) = \lambda||\boldsymbol{\alpha}||_0$ for $\mathbf{x} = \mathbf{D}\boldsymbol{\alpha}$

$$\underbrace{\begin{pmatrix} \\ \mathbf{x} \\ \\ \end{pmatrix}}_{\mathbf{x}\in\mathbb{R}^N} = \underbrace{\left(\begin{array}{c|c|c|c} & & & \\ \mathbf{d}_1 & \mathbf{d}_2 & \cdots & \mathbf{d}_k \\ & & & \end{array}\right)}_{\mathbf{D}\in\mathbb{R}^{N\times k}} \underbrace{\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_k \end{pmatrix}}_{\boldsymbol{\alpha}\in\mathbb{R}^k,\text{sparse}}$$

Designed sparse representations

[Haar 1909], [Zweig, Morlet, Grossman ∼70s], [Meyer, Mallat, Daubechies, Coifman, Donoho, Candes ∼80s-today]...

Which dictionary to choose?

- Wavelets
- Curvelets
- Wedgelets
- Bandlets
- ...lets

Learned sparse representations

[Fields & Olshausen '96], [MOD: Engan et. al '99],[Lewicki & Sejnowski '00],[K-SVD: Aharon, Elad & Bruckstein '04 '05],[FoE: Roth & Black '05],[Lee et al. '06],[Neural nets: Lecun, Hinton ∼90s-today.]

Learned dictionaries of patches

$$\min_{\boldsymbol{\alpha}_i, \mathbf{D} \in \mathcal{C}} \sum_i \underbrace{||\mathbf{x}_i - \mathbf{D}\boldsymbol{\alpha}_i||_2^2}_{\text{reconstruction}} + \underbrace{\lambda \phi(\boldsymbol{\alpha}_i)}_{\text{sparsity}}$$

- $\phi(\boldsymbol{\alpha}) = ||\boldsymbol{\alpha}||_0$ ("ℓ_0 pseudo-norm")
- $\phi(\boldsymbol{\alpha}) = ||\boldsymbol{\alpha}||_1$ (ℓ_1 norm)

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MOD: [Engan et. al '99]

$$\{\mathbf{D}, \boldsymbol{\alpha}\} = \arg\min_{\mathbf{D}\in\mathcal{C},\boldsymbol{\alpha}} \sum_{i=1}^{P} ||\mathbf{x}_i - \mathbf{D}\boldsymbol{\alpha}_i||_2^2 + \mu_i||\boldsymbol{\alpha}_i||_0$$

Initialization of $\mathbf{D}$ → ex: DCT

Sparse Coding → Fix $\mathbf{D}$ and $\forall i \in 1 \cdots P$,

$$\{\boldsymbol{\alpha}_i\} \approx \arg\min_{\boldsymbol{\alpha}} ||\mathbf{x}_i - \mathbf{D}\boldsymbol{\alpha}||_2^2 + \mu_i||\boldsymbol{\alpha}||_0$$

using a Greedy approach

Dictionary Update → $$\{\mathbf{D}\} = \arg\min_{\mathbf{D}\in\mathcal{C}} \sum_i ||\mathbf{x}_i - \mathbf{D}\boldsymbol{\alpha}_i||_2^2$$

Sparse representations for image restoration

K-SVD: [Elad & Aharon ('06)]

$$\{\mathbf{D}, \boldsymbol{\alpha}\} = \arg\min_{\mathbf{D}\in\mathcal{C},\boldsymbol{\alpha}} \sum_{i=1}^{P} ||\mathbf{x}_i - \mathbf{D}\boldsymbol{\alpha}_i||_2^2 + \mu_i||\boldsymbol{\alpha}_i||_0$$

Initialization of D → ex: DCT

Sparse Coding → Fix $\mathbf{D}$ and $\forall i \in 1\cdots P$,

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using a Greedy approach

Dictionary Update → Sequantially, $\forall j = 1\cdots K$: Fix all $\mathbf{d}_{l\neq j}$, and minimize the reconstruction error respect to $\mathbf{d}_j$ and the non-zeros $\boldsymbol{\alpha}_i(j)$,

Sparse representations for image restoration

ℓ_1: Lee et al. '06]

$$\{\mathbf{D}, \boldsymbol{\alpha}\} = \arg\min_{\mathbf{D}\in\mathcal{C},\boldsymbol{\alpha}} \sum_{i=1}^{P} ||\mathbf{x}_i - \mathbf{D}\boldsymbol{\alpha}_i||_2^2 + \mu_i||\boldsymbol{\alpha}_i||_1$$

Initialization of $\mathbf{D}$ → ex: DCT

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$$\{\boldsymbol{\alpha}_i\} = \arg\min_{\boldsymbol{\alpha}} ||\mathbf{x}_i - \mathbf{D}\boldsymbol{\alpha}||_2^2 + \mu_i||\boldsymbol{\alpha}||_1$$

using LARS, coordinate descent,. . . ...

Dictionary Update →

$$\{\mathbf{D}\} = \arg\min_{\mathbf{D}\in\mathcal{C}} \sum_i ||\mathbf{x}_i - \mathbf{D}\boldsymbol{\alpha}_i||_2^2$$

Sparse representations for image restoration

K-SVD: [Elad & Aharon ('06)]

Key ideas for denoising

- Consider each patch of size $n \times n$ ($n = 8$) in the image, including overlaps.
- learn the dictionary on the corrupted image.
- the Sparse Coding retrieve a sparse approximation of the *noisy* patches.
- Average the approximation of each patch to reconstruct the full image.

Sparse representations for image restoration

K-SVD: [Elad & Aharon ('06)]

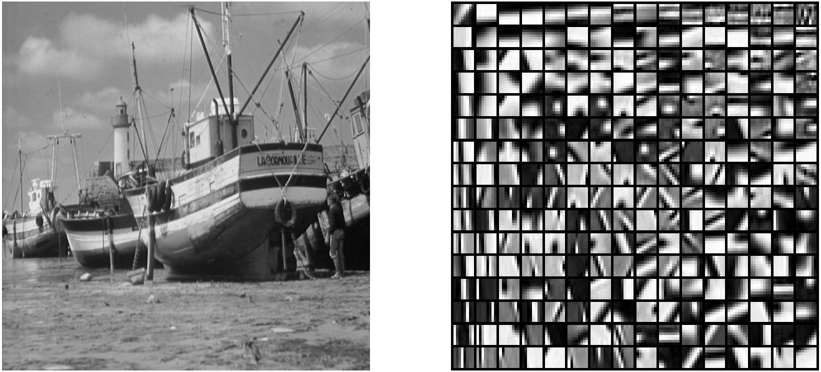


Figure: Dictionary trained on a noisy version of the image boat.

The multiscale extension
[Mairal, Sapiro & Elad ('07)]

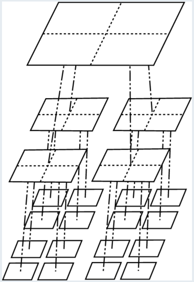
Different ways of "thinking" multiscale

- Image Pyramids (Gaussian, Laplacian) ?
- Working with different sizes of patches at full resolution

The multiscale extension
[Mairal, Sapiro & Elad ('07)]

The key changes

- A Quadtree for each patch
- One dictionary per scale
- multiscale decomposition of each patch



The multiscale extension
[Mairal, Sapiro & Elad ('07)]

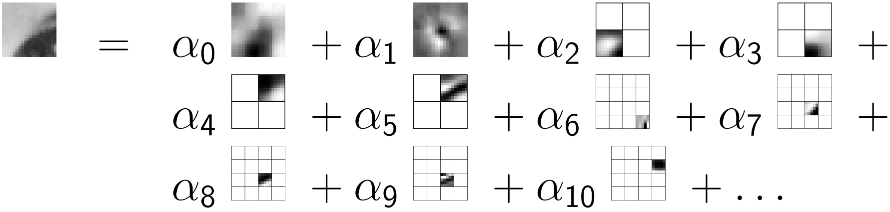


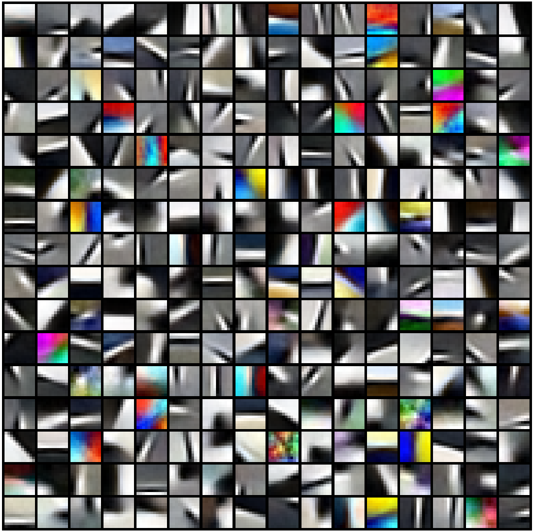
Figure: Possible decomposition of a 20×20 patch with a 3-scales dictionary.

The multiscale extension
[Mairal, Sapiro & Elad ('07)]

Figure: On the left: original image. In the middle, image corrupted ($\sigma = 15$). On the right, the result with 3 scales (PSNR=32.01dB)

Sparse representations for image restoration

Color denoising, [Mairal, Sapiro & Elad ('08)]



- Most of the atoms are grey!
- Color sparse approximations suffers from color artefacts.
- Average color should be taken into account during sparse approximation:

$$< \mathbf{x_1}, \mathbf{x_2} >_\gamma = \mathbf{x_1}^T \mathbf{x_2} + \gamma < \bar{\mathbf{x}}_1, \bar{\mathbf{x}}_2 >$$

Sparse representations for image restoration

Color denoising, [Mairal, Sapiro & Elad ('08)]

Figure: Denoising result for $\sigma = 25$ and 2 scales.

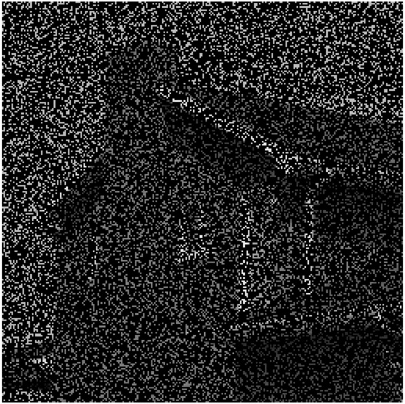
Sparse representations for image restoration

Inpainting, [Mairal, Sapiro & Elad ('08)]

$$\min_{\mathbf{D},\boldsymbol{\alpha}} \sum_i \sum_i ||\boldsymbol{\beta}_i \otimes (\mathbf{y}_i - \mathbf{D}\boldsymbol{\alpha}_i)||_2^2 + \lambda_i ||\boldsymbol{\alpha}_i||_0$$

Sparse representations for image restoration

Inpainting, [Mairal, Sapiro & Elad ('08)]



Restored image on the right.

Sparse representations for image restoration

Inpainting, [Mairal, Sapiro & Elad ('08)]

Sparse representations for image restoration
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Sparse representations for image restoration

Inpainting, [Mairal, Sapiro & Elad ('08)]

Sparse representations for video restoration

Denoising, [Protter & Elad ('08)]

Key ideas for video processing

- Using a 3D dictionary.
- Processing of many frames at the same time.
- Dictionary propagation.

Sparse representations for image restoration

Inpainting, [Mairal, Sapiro & Elad ('08)]

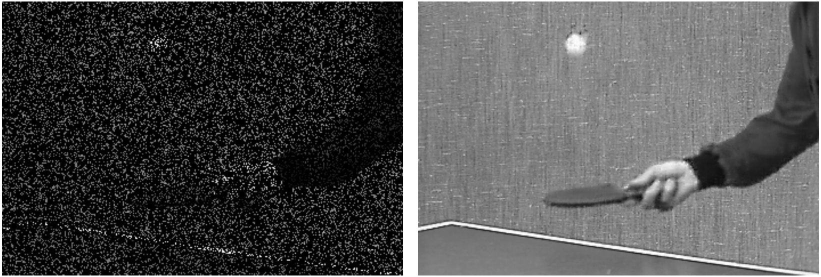
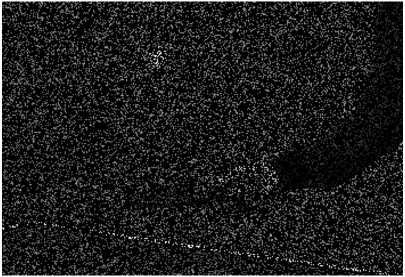
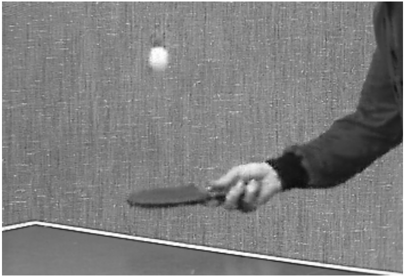


Figure: Inpainting results with two scales.

Sparse representations for image restoration
Inpainting, [Mairal, Sapiro & Elad ('08)]





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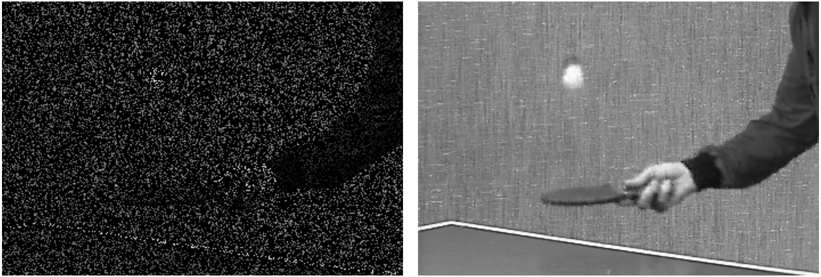

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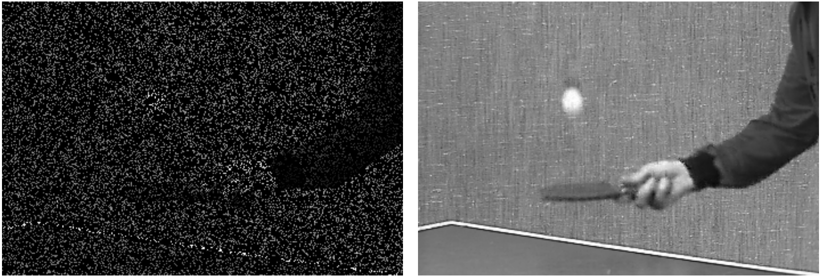

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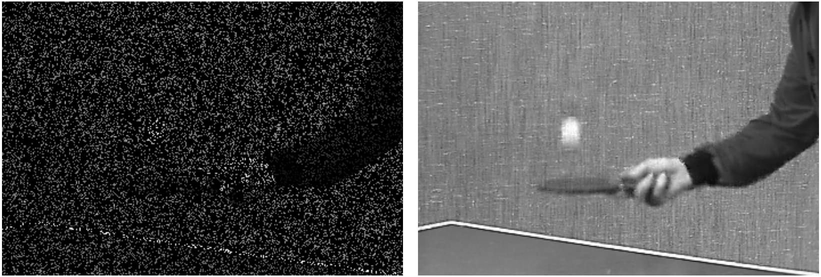
Sparse representations for image restoration

Inpainting, [Mairal, Sapiro & Elad ('08)]



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Sparse representations for image restoration

Color Video denoising, [Mairal, Sapiro & Elad ('08)]

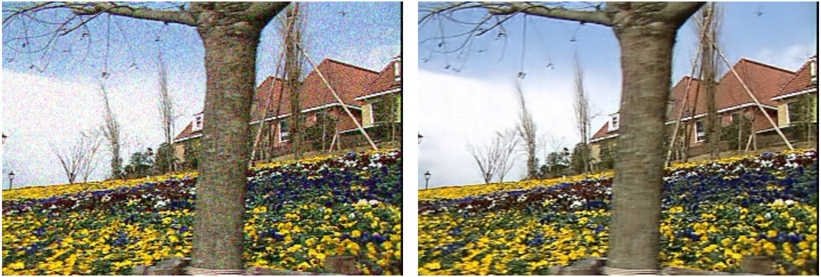
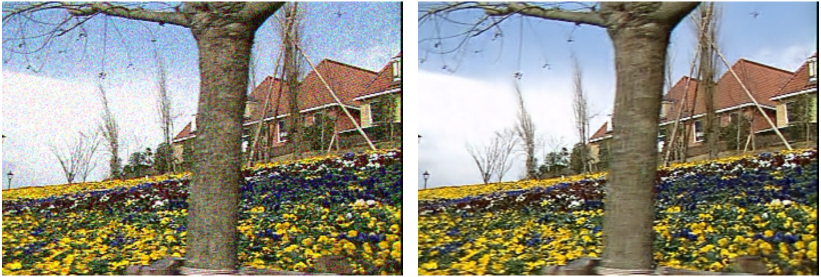


Figure: Denoising results with two scales. $\sigma = 25$

Sparse representations for image restoration
Color Video denoising, [Mairal, Sapiro & Elad ('08)]



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Sparse representations for image restoration

Color Video denoising, [Mairal, Sapiro & Elad ('08)]

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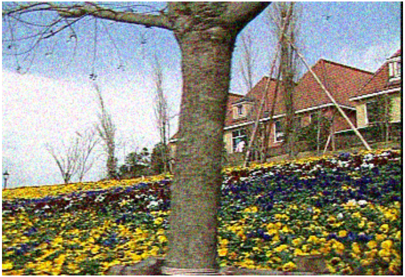

Figure: Denoising results with two scales. $\sigma = 25$

More information at

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