Sparse Learned Representations for Image Restoration

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What this talk is about

- The dictionary learning paradigm.
- Various formulations for image and video processing.
- A fast online dictionary learning algorithm.
- Processing raw images from digital cameras.
1. Sparse representations for image denoising
2. Formulations for image and video processing
3. Online Dictionary Learning
4. Raw image processing
1. Sparse representations for image denoising

2. Formulations for image and video processing

3. Online Dictionary Learning

4. Raw image processing
Sparse representations for image denoising

\[
\mathbf{y} = \mathbf{x}_{\text{orig}} + \mathbf{w}
\]

- \(\mathbf{y}\): measurements
- \(\mathbf{x}_{\text{orig}}\): original image
- \(\mathbf{w}\): white Gaussian noise
Sparse representations for image denoising

Energy minimization problem - MAP estimation:

\[ E(x) = \| y - x \|^2_2 + Pr(x) \]

relation to measurements

prior
Sparse representations for image denoising

Some classical priors

- Smoothness $\lambda \| \mathbf{Lx} \|_2^2$
- Total variation $\lambda \| \nabla \mathbf{x} \|_2^2$
- Wavelet sparsity $\lambda \| \mathbf{Wx} \|_1$
- ...
Sparse representations for image denoising

\[ Pr(x) = \lambda \|\alpha\|_0 \text{ for } x = D\alpha \]

\[
\begin{pmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_k
\end{pmatrix}
\in \mathbb{R}^k, \text{sparse}
\]
Sparse representations for image denoising
[Haar 1909], [Zweig, Morlet, Grossman ∼70s], [Meyer, Mallat, Daubechies, Coifman, Donoho, Candes ∼80s-today]...

Which dictionary to choose?
- Wavelets
- Curvelets
- Wedgelets
- Bandlets
- ...lets
Sparse representations for image denoising
[Fields & Olshausen '96], [MOD: Engan et. al '99],[Lewicki & Sejnowski '00],[K-SVD: Aharon, Elad & Bruckstein '04 '05],[FoE: Roth & Black '05],[Lee et al. '06],[Neural nets: Lecun, Hinton ∼90s-today.]

Learned dictionaries of patches

$$\min_{\alpha_i, D \in \mathcal{C}} \sum_i \| x_i - D\alpha_i \|^2_2 + \lambda \phi(\alpha_i)$$

- reconstruction
- sparsity

$$\phi(\alpha) = \|\alpha\|_0$$ ("$\ell_0$ pseudo-norm")

$$\phi(\alpha) = \|\alpha\|_1$$ ($\ell_1$ norm)
Sparse representations for image denoising
K-SVD: [Elad & Aharon (’06)]

Key ideas for denoising

- Consider each patch of size $\sqrt{m} \times \sqrt{m}$ ($\sqrt{m} = 8$) in the image, including overlaps.
- Learn the dictionary on the corrupted image.
- The Sparse Coding retrieve a sparse approximation of the noisy patches.
- Average the approximation of each patch to reconstruct the full image.
<table>
<thead>
<tr>
<th>$\ell_0$ vs $\ell_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use $\ell_1$ for learning the dictionary!</td>
</tr>
<tr>
<td>Use $\ell_0$ for reconstructing the image!</td>
</tr>
</tbody>
</table>
Sparse representations for image denoising

K-SVD: [Elad & Aharon ('06)]

Figure: Dictionary trained on a noisy version of the image boat using K-SVD.
1. Sparse representations for image denoising
2. Formulations for image and video processing
3. Online Dictionary Learning
4. Raw image processing
A multiscale extension
[Mairal, Sapiro & Elad (’07)]

The key changes

- A Quadtree for each patch
- One dictionary per scale
- Multiscale decomposition of each patch
A multiscale extension

[Mairal, Sapiro & Elad ('07)]

Figure: On the left: original image. In the middle, image corrupted ($\sigma = 15$). On the right, the result with 3 scales (PSNR=32.01dB)
Preliminary results

[Mairal, Bach, Ponce, Sapiro & Zisserman ('09)]
Preliminary results
[Mairal, Bach, Ponce, Sapiro & Zisserman (’09)]
Sparse representations for image restoration
Color denoising, [Mairal, Sapiro & Elad (’08)]
Most of the atoms are grey!

Color sparse approximations suffers from color artefacts.

Average color should be taken into account during sparse approximation!
Sparse representations for image restoration
Color denoising, [Mairal, Sapiro & Elad (’08)]

Figure: Denoising result for $\sigma = 25$ and 2 scales.
Sparse representations for image restoration

Inpainting, [Mairal, Sapiro & Elad ('08)]

\[
\min_{\alpha_i, D \in \mathcal{C}} \sum_i \left\| \beta_i (x_i - D \alpha_i) \right\|_2^2 + \lambda \phi(\alpha_i)
\]

reconstruction \hspace{1cm} \text{sparsity}
Sparse representations for image restoration
Inpainting, [Mairal, Sapiro & Elad ('08)]

Restored image on the right.
Sparse representations for image restoration
Inpainting, [Mairal, Sapiro & Elad ('08)]
Since 1699, when French explorers landed at the great bend of the Mississippi River and celebrated the first Mardi Gras in North America, New Orleans has brewed a fascinating melange of cultures. It was French, then Spanish, then French again, then sold to the United States. Through all these years, and even into the 1900s, others arrived from everywhere: Acadians (Cajuns), Africans, indige–
Sparse representations for image restoration
Inpainting, [Mairal, Sapiro & Elad ('08)]
Figure: Inpainting results with two scales.
Sparse representations for image restoration

Inpainting, [Mairal, Sapiro & Elad ('08)]

Figure: Inpainting results with two scales.
Sparse representations for image restoration
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Sparse representations for image restoration
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Figure: Inpainting results with two scales.
Sparse representations for image restoration

Color Video denoising, [Mairal, Sapiro & Elad ('08)]

Figure: Denoising results with two scales. $\sigma = 25$
Sparse representations for image restoration
Color Video denoising, [Mairal, Sapiro & Elad (’08)]

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Sparse representations for image denoising

Formulations for image and video processing

Online Dictionary Learning

Raw image processing
Online Dictionary Learning
[Mairal, Bach, Ponce & Sapiro (’09)]

Classical formulation for dictionary learning

\[
\min_{D \in \mathcal{C}} f_n(D) = \min_{D \in \mathcal{C}} \frac{1}{n} \sum_{i=1}^{n} l(x_i, D),
\]

where

\[
l(x, D) = \min_{\alpha \in \mathbb{R}^k} \frac{1}{2} \|x_i - D\alpha_i\|_2^2 + \lambda \|\alpha\|_1.
\]
Which formulation are we interested in?

$$\min_{D \in C} f(D) = \mathbb{E}_x[l(x, D)] = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} l(x, D).$$
Online learning can

- handle potentially infinite datasets.
- be dramatically faster than batch algorithms.
- adapt to dynamic training sets.
Online Dictionary Learning
[Mairal, Bach, Ponce & Sapiro ('09)]

1: Initialization of $D_0$,
2: for $t = 1$ to $T$ do
3: Draw $x_t$.
4: Sparse coding: compute using LARS
   $\alpha_t \leftarrow \text{arg min}_{\alpha \in \mathbb{R}^k} \frac{1}{2} \| x_t - D_{t-1} \alpha \|_2^2 + \lambda \| \alpha \|_1$.
5: Compute $D_t$ using $D_{t-1}$ as warm restart,
   $D_t \leftarrow \text{arg min}_{D \in \mathcal{C}} \frac{1}{t} \sum_{i=1}^{t} \frac{1}{2} \| x_i - D \alpha_i \|_2^2 + \lambda \| \alpha_i \|_1$.
6: end for
7: return $D_T$ (learned dictionary).
Online Dictionary Learning
[Mairal, Bach, Ponce & Sapiro (’09)]

$m = 12 \times 12$ color patches, $k = 512$. 
Online Dictionary Learning

Inpainting a 12Mpixels photograph
A few simple extensions

- sparse dictionaries.
- non-negative matrix factorization.
- sparse PCA.
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What is a raw image?

+ noise
Raw image processing

The raw image processing pipeline

1. Denoising of the mosaick.
2. Demosaicking.
3. Color conversion to sRGB.
Preliminary results
[Mairal, Bach, Ponce, Sapiro & Zisserman ('09)]
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Learned sparse representations

- can adapt to various type of data.
- lead to state-of-the-art results for several tasks.
- are computationally cheap thanks to online learning.