Fisher kernels with application to image representation

Advanced Learning Models 2017-2018

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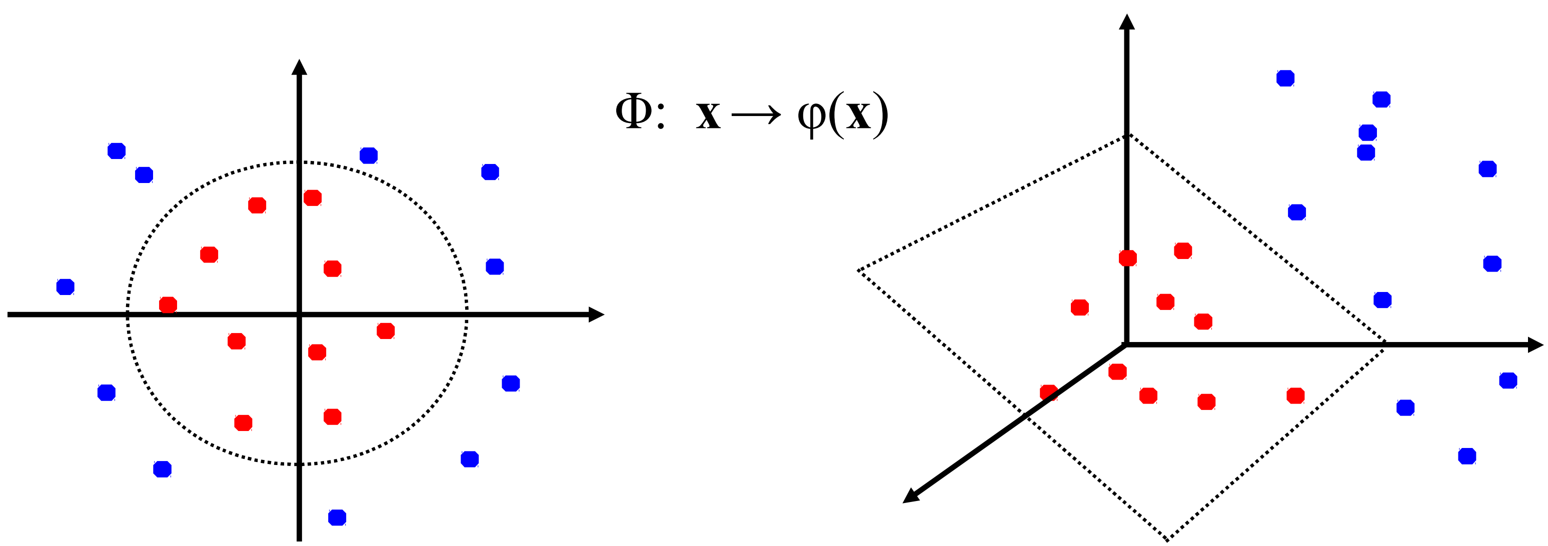
A brief recap on kernel methods

- A way to achieve non-linear classification (or other data analysis) by using a kernel that computes inner products of data after non-linear transformation
  - Given the transformation, we can derive the kernel function.
- Conversely, if a kernel is positive definite, it is known to compute a dot-product in a (not necessarily finite dimensional) feature space
  - Given the kernel, we can determine the feature mapping function.

$$k(x_1, x_2) = \langle \phi(x_1), \phi(x_2) \rangle$$



A brief recap on kernel methods

- Most often we start with data in a vector space, and map it to another feature space to allow for non-linear classification in the original space, using linear classification in the feature space
- Kernels can also be used to represent non-vectorial data, and to make them amenable to linear classification (or other linear data analysis) techniques
- For example, suppose we want to classify sets of points in a vector space, where the size of each set may vary

$$X = \{x_1, x_2, \ldots, x_N\} \quad \text{with} \quad x_i \in R^d$$

- We can define a representation of sets by concatenating the mean and variance of the set in each dimension

$$\phi(X) = \begin{pmatrix} \text{mean}(X) \\ \text{var}(X) \end{pmatrix}$$

  - Fixed size representation of sets in 2d dimensions
  - Use kernel to compare different sets:

$$k(X_1, X_2) = \langle \phi(X_1), \phi(X_2) \rangle$$

Fisher kernels

- Motivated by the need to represent variably sized objects in a vector space, such as sequences, sets, trees, graphs, etc., such that they become amenable to be used with linear classifiers, and other data analysis tools

- A generic method to define kernels over arbitrary data types based on statistical model of the items we want to represent

$$p(x;\theta), \quad x \in X, \quad \theta \in R^D$$

- Parameters and/or structure of the model p(x) estimated from data
  - Typically in unsupervised manner
- Automatic data-driven configuration of kernel instead of manual design
  - Kernel typically used for supervised task

[Jaakkola & Haussler, “Exploiting generative models in discriminative classifiers”,In Advances in Neural Information Processing Systems 11, 1998.]

Fisher kernels

- Given a generative data model $p(x;\theta),\ \ x\in X,\ \ \theta\in R^D$
- Represent data x in X by means of the gradient of the data log-likelihood, or "Fisher score":

$$g(x)=\nabla_\theta \ln p(x),$$
$$g(x)\in R^D$$

- Define a kernel over X by taking the scaled inner product between the Fisher score vectors:

$$k(x,y)=g(x)^T F^{-1} g(y)$$

- Where F is the Fisher information matrix F:

$$F=\boldsymbol{E}_{p(x)}\left[g(x)g(x)^T\right]$$

- F is positive definite since

$$\alpha^T F\alpha=\boldsymbol{E}_{p(x)}\left[\left(g(x)^T\alpha\right)^2\right]>0$$

Fisher kernels

- The Fisher score has zero mean under the generative model

$$\begin{aligned}
E_{p(x)}[g(x)] &= \int_x p(x) \frac{\partial}{\partial \theta} \ln p(x) \\
&= \int_x p(x) \frac{1}{p(x)} \frac{\partial}{\partial \theta} p(x) \\
&= \int_x \frac{\partial}{\partial \theta} p(x) \\
&= \frac{\partial}{\partial \theta} \int_x p(x) \\
&= \frac{\partial}{\partial \theta} 1 \\
&= 0
\end{aligned}$$

- Therefore, the Fisher information matrix is the covariance matrix of the Fisher score under the generative model

$$F = \boldsymbol{E}_{p(x)}\left[g(x) g(x)^T \right]$$

Fisher vector

- Since F is positive definite we can decompose its inverse as

$$F^{-1} = L^T L$$

- Therefore, we can write the kernel as

$$k(x_i, x_j) = g(x_i)^T F^{-1} g(x_j) = \phi(x_i)^T \phi(x_j)$$

  - Where phi is known as the **Fisher vector**

$$\phi(x_i) = L\, g(x_i)$$

- From this explicit finite-dimensional data embedding it follows immediately that the Fisher kernel is a positive-semidefinite

- Since F is covariance of Fisher score, normalization by L makes the Fisher vector have unit covariance matrix under p(x)

Normalization with inverse Fisher information matrix

- Gradient of log-likelihood w.r.t. parameters $g(x) = \nabla_\theta \ln p(x)$
- Fisher information matrix $F_\theta = \int g(x) g(x)^T p(x) dx$
- Normalized Fisher kernel $k(x_1, x_2) = g(x_1)^T F_\theta^{-1} g(x_2)$
  - Renders Fisher kernel invariant for parametrization

- Consider different parametrization given by some invertible function $\lambda = f(\theta)$
- Jacobian matrix relating the parametrizations $[J]_{ij} = \frac{\partial \theta_j}{\partial \lambda_i}$
- Gradient of log-likelihood w.r.t. new parameters

$$h(x) = \nabla_\lambda \ln p(x) = J \nabla_\theta \ln p(x) = J g(x)$$

- Fisher information matrix $F_\lambda = \int h(x) h(x)^T p(x) dx = J F_\theta J^T$
- Normalized Fisher kernel

$$\begin{aligned} h(x_1)^T F_\lambda^{-1} h(x_2) &= g(x_1)^T J^T (J F_\theta J^T)^{-1} J g(x_2) \\ &= g(x_1)^T J^T J^{-T} F_\theta^{-1} J^{-1} J g(x_2) \\ &= g(x_1)^T F_\theta^{-1} g(x_2) \\ &= k(x_1, x_2) \end{aligned}$$

Fisher kernels: example with Gaussian data model

- Let lambda be the inverse variance, i.e. precision, parameter

$$p(x) = N(x;\mu,\lambda) = \sqrt{\lambda/(2\pi)} \exp\left[-\frac{1}{2}\lambda(x-\mu)^2\right]$$

$$\ln p(x) = \frac{1}{2}\ln\lambda - \frac{1}{2}\ln(2\pi) - \frac{1}{2}\lambda(x-\mu)^2$$

$$\theta = (\mu,\lambda)^T$$

- The partial derivatives are found to be

$$\frac{\partial \ln p(x)}{\partial \mu} = \lambda(x-\mu) \qquad \frac{\partial \ln p(x)}{\partial \lambda} = \frac{1}{2}\left[\lambda^{-1} - (x-\mu)^2\right]$$

Fisher kernels: example with Gaussian data model

- Now suppose an i.i.d. data model over a set of data points

$$p(x) = N(x; \mu, \lambda) = \sqrt{\lambda/(2\pi)} \exp\left[-\frac{1}{2}\lambda(x-\mu)^2\right]$$

$$p(X) = p(x_{1,\ldots}, x_N) = \prod_{i=1}^{N} p(x_i)$$

- Then the Fisher vector is given by the sum of Fisher vectors of the points
  - Encodes the discrepancy in the first and second order moment of the data w.r.t. those of the model

$$\phi(X) = \sum_{i=1}^{N} \phi(x_i) = N \begin{pmatrix} (\hat{\mu}-\mu)/\sigma \\ (\sigma^2 - \hat{\sigma}^2)/(\sigma^2\sqrt{2}) \end{pmatrix}$$

  - Where

$$\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad \hat{\sigma}^2 = \frac{1}{N}\sum_{i=1}^{N} (x_i - \mu)^2$$

Fisher kernels – relation to generative classification

- Suppose we make use of generative model for classification via Bayes' rule
  - Where x is the data to be classified, and y is the discrete class label

$$p(y|x) = p(x|y)\, p(y) / p(x),$$
$$p(x) = \sum_{k=1}^{K} p(y=k)\, p(x|y=k)$$

and

$$p(x|y) = p(x;\theta_y),$$
$$p(y=k) = \pi_k = \frac{\exp(\alpha_k)}{\sum_{k'=1}^{K} \exp(\alpha_{k'})}$$

- Classification with the Fisher kernel obtained using the marginal distribution p(x) is at least as powerful as classification with Bayes' rule.
- This becomes useful when the class conditional models are poorly estimated, either due to bias or variance type of errors.
- In practice often used without class-conditional models, but direct generative model for the marginal distribution on X.

Fisher kernels – relation to generative classification

- Consider the Fisher score vector with respect to the marginal distribution on X

$$\begin{aligned}\nabla_\theta \ln p(x) &= \frac{1}{p(x)} \nabla_\theta \sum_{k=1}^{K} p(x, y=k) \\ &= \frac{1}{p(x)} \sum_{k=1}^{K} p(x, y=k) \nabla_\theta \ln p(x, y=k) \\ &= \sum_{k=1}^{K} p(y=k|x) \left[\nabla_\theta \ln p(y=k) + \nabla_\theta \ln p(x|y=k) \right]\end{aligned}$$

- In particular for the alpha that model the class prior probabilities we have

$$\frac{\partial \ln p(x)}{\partial \alpha_k} = p(y=k|x) - \pi_k$$

Fisher kernels – relation to generative classification

- First K elements in Fisher score given by class posteriors minus a constant

$$g(x) = \nabla_\theta \ln p(x) = \left(p(y=1|x) - \pi_{1,} \ldots, p(y=K|x) - \pi_K, \ldots \right)$$

- Consider discriminative multi-class classifier, for the k-th class
  - Let the weight vector be zero, except for the k-th position where it is one
  - Let the bias term be equal to the prior probability of that class
- Then

$$f_k(x) = w_k^T g(x) + b_k = p(y=k|x)$$

and thus

$$\operatorname{argmax}_k \ f_k(x) = \operatorname{argmax}_k \ p(y=k|x)$$

- Thus the Fisher kernel based classifier can implement classification via Bayes' rule, and generalizes it to other classification functions.

Application in visual object recognition

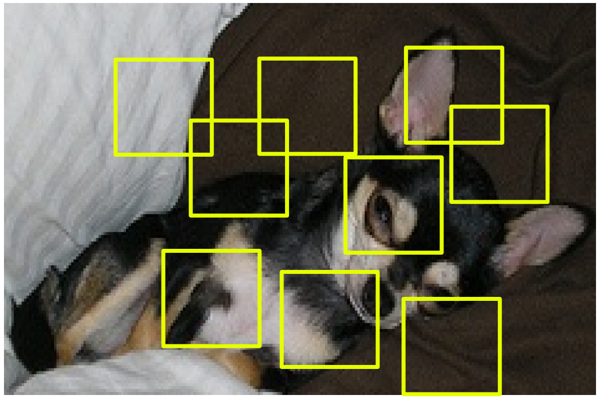
- A number of challenging factors
- Intra-class appearance variation
  - Category contains many instances
  - Objects deformation due to pose
  - Sub-categories: boat = ferry + yacht +...

- Scene composition
  - Heavy occlusions: e.g. tables and chairs
  - Clutter: coincidental image content present

- Imaging conditions
  - viewpoint, scale, illumination

Representing images as “bags of features”

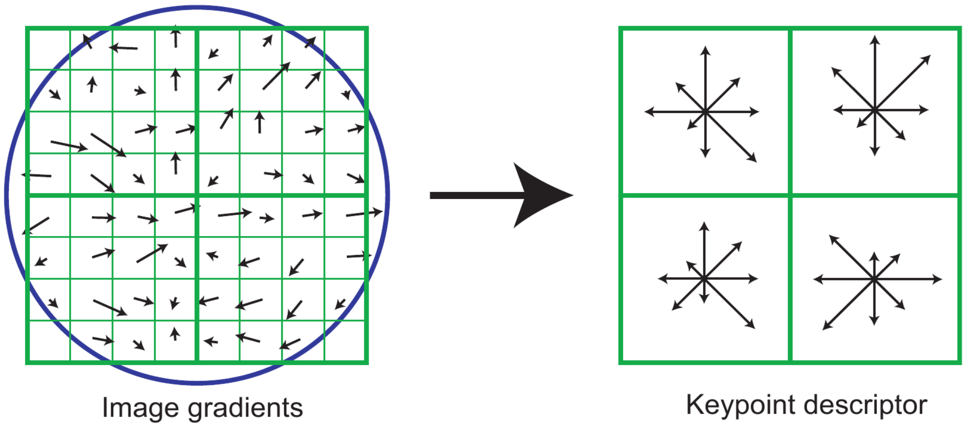
- Global rigid representation likely to be affected by nuisance factors such as deformation, (self-)occlusion, clutter, etc.
- Instead consider local image regions, or “patches”, on which some representation is computed that is (partially) invariant to imaging conditions such as viewpoint, illumination, scale, etc.
    - Local patterns more likely to be preserved, or at least some of them
- Patch extraction and description stage
    - Patch sampling from image on dense multi-scale grid, or interest points
    - Descriptor computation: SIFT, HOG, LBP, color names, …
- Set of local descriptors characterizes the image (or video, or speech, or ...)
- Feature aggregation stage
    - Global image signature computed
    - Can be classified or used for matching
- See [Sivic & Zisseman, ICCV’03]



Local descriptor based image representations

- SIFT patch description most popular
  - 4x4 spatial grid
  - 8 bin orientation histogram [Lowe, IJCV 2004]

$$X=\{x_1,\ldots,x_N\}$$

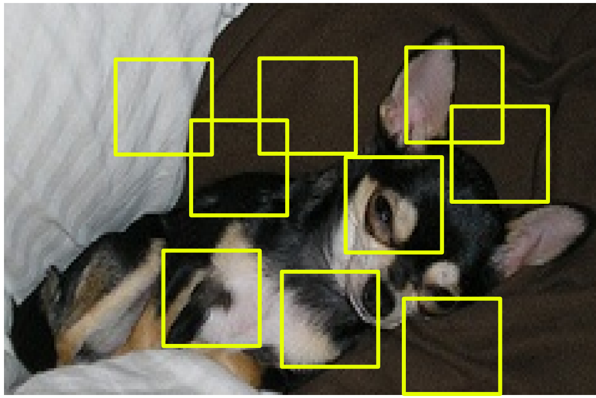


- Coding stage: embed local descriptors, typically in higher dimensional space
  - For example: 1-hot coding of the index of the nearest cluster center

$$\phi(x_i)$$

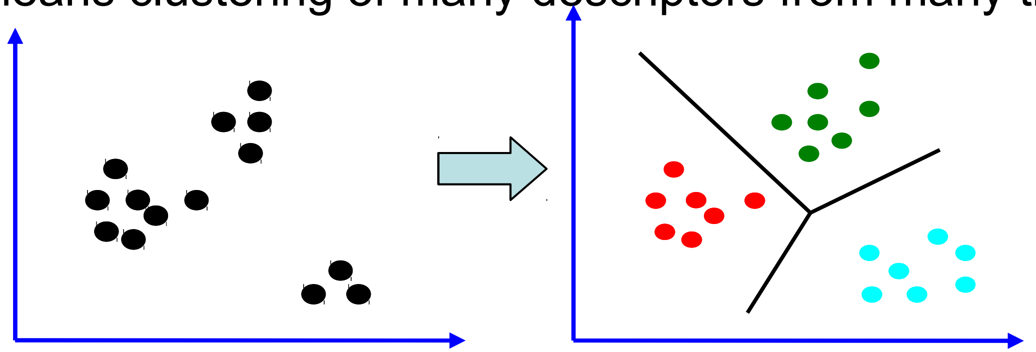
- Pooling stage: aggregate per-patch embeddings
  - For example: sum pooling

$$\Phi(X)=\sum_{i=1}^{N}\phi(x_i)$$



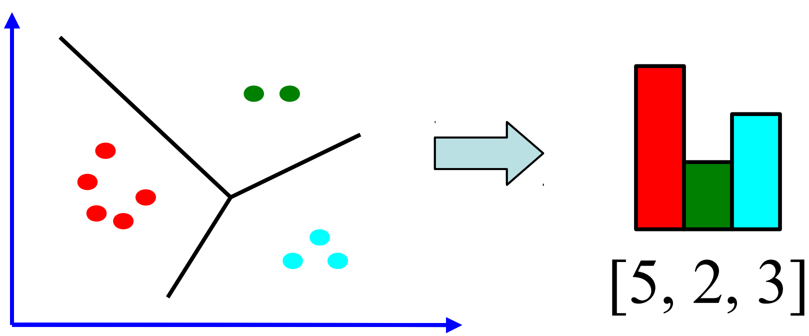
The “bag of visual words” representation

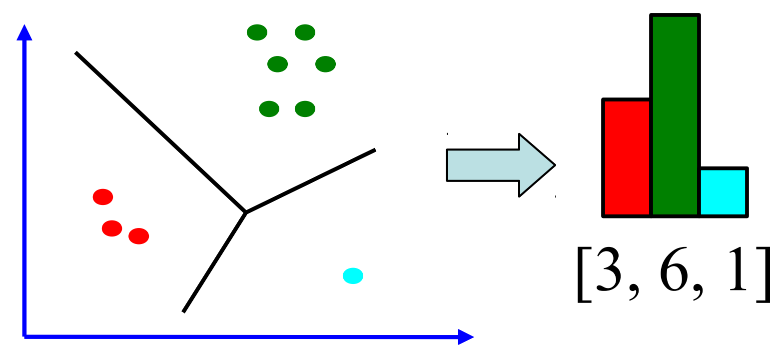
- Offline k-means clustering of many descriptors from many training images



- Encoding a new image:
  - Compute local descriptors, assign to cluster
  - Count histogram of descriptors in each cluster
- Sum pooling of “1-hot encoding” of local descriptors

$$\phi(x_i) = [0, \ldots, 0, 1, 0, \ldots, 0] \qquad h = \sum_i \phi(x_i)$$





Examples of clusters of local image descriptors

Airplanes

Motorbikes

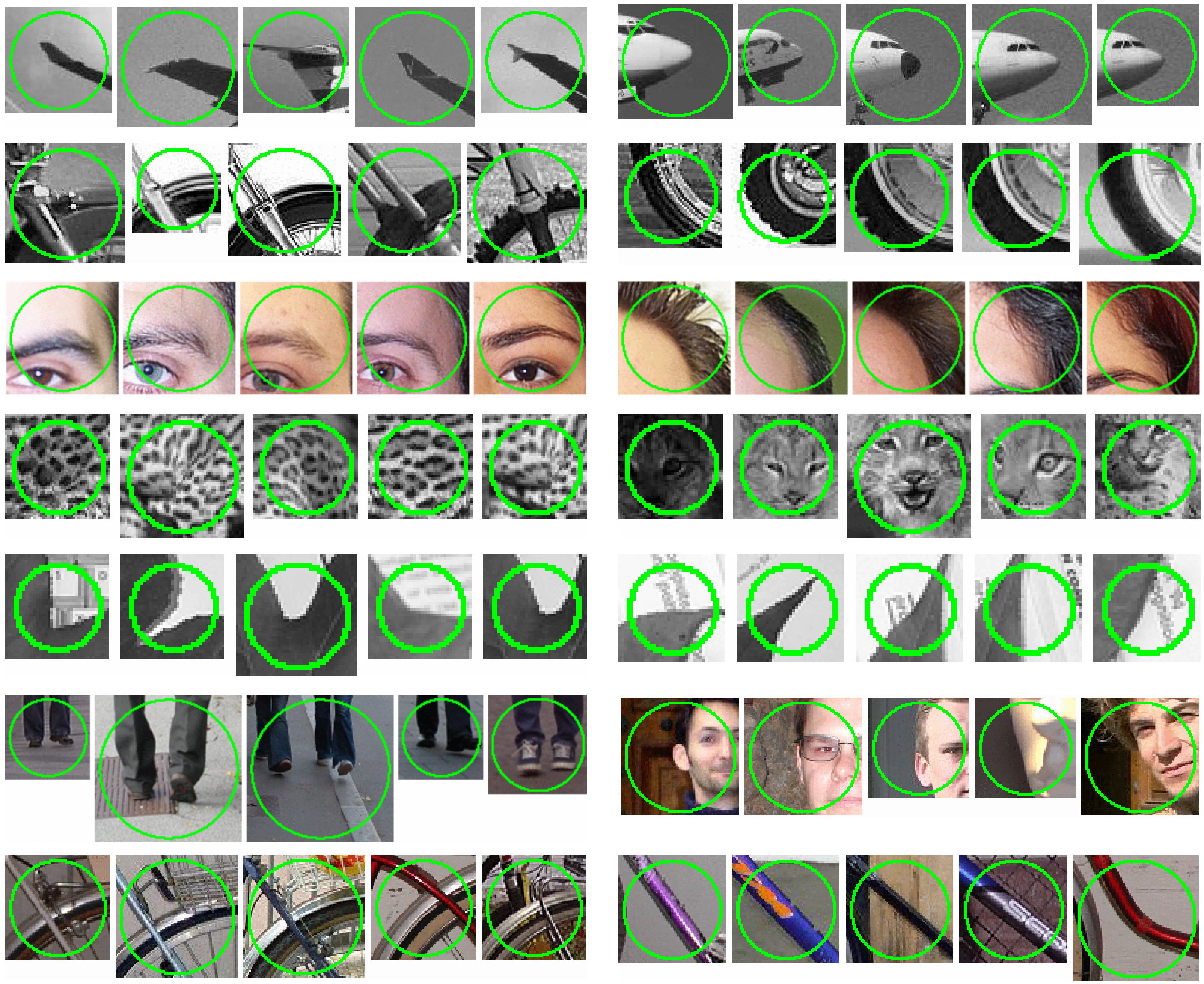
Faces

Wild Cats

Leafs

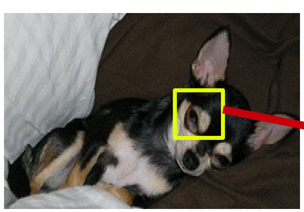
People

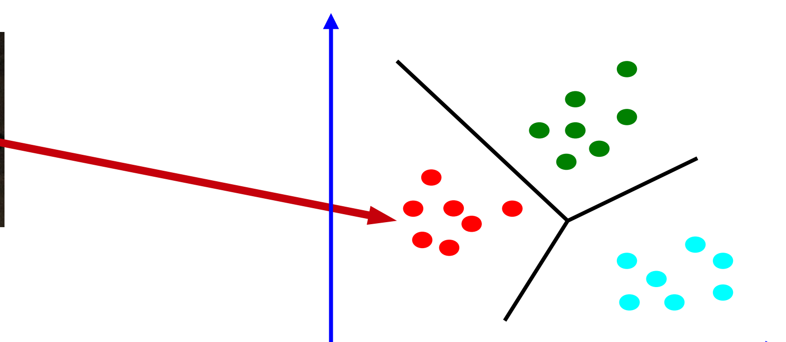
Bikes

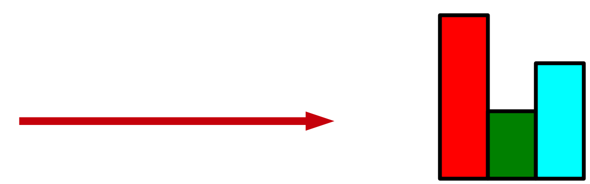


Application of FV for bag-of-words image-representation

- Bag of word (BoW) representation
  - Map every descriptor to a cluster / visual word index $w_i \in \{1, \ldots, K\}$
- Model visual word indices with i.i.d. multinomial $p(w_i = k) = \frac{\exp \alpha_k}{\sum_{k'} \exp \alpha_{k'}} = \pi_k$
  - Likelihood of N i.i.d. indices: $p(w_{1:N}) = \prod_{i=1}^{N} p(w_i)$
  - Fisher vector given by gradient $\frac{\partial \ln p(w_{1:N})}{\partial \alpha_k} = \sum_{i=1}^{N} \frac{\partial \ln p(w_i)}{\partial \alpha_k} = h_k - N \pi_k$
    - i.e. BoW histogram + constant



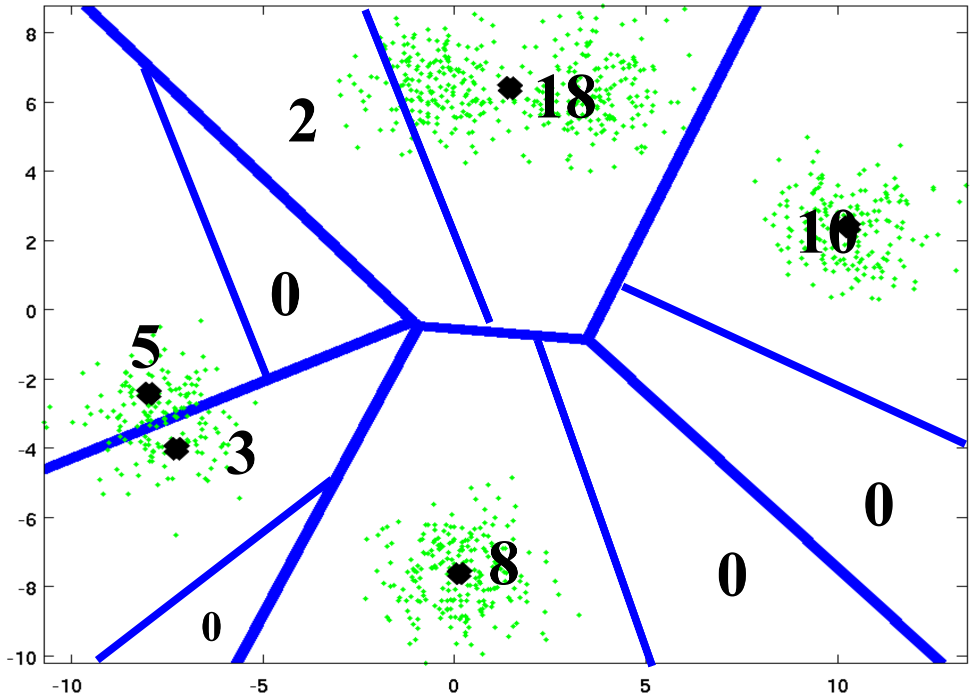




Fisher vector GMM representation: Motivation

- Suppose we want to refine a given visual vocabulary to obtain a richer image representation

- Bag-of-word histogram stores # patches assigned to each word
  - Need more words to refine the representation
  - But this directly increases the computational cost
  - And leads to many empty bins: redundancy

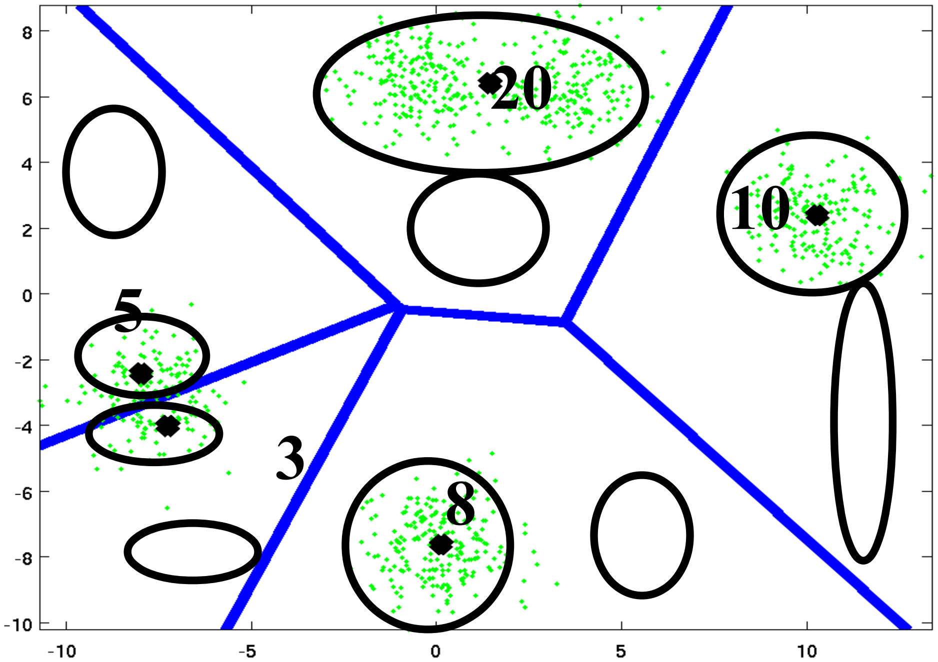


Fisher vector representation in a nutshell

- Instead, the Fisher Vector for GMM also records the mean and variance of the points per dimension in each cell
  - More information for same # visual words
  - Does not increase computational time significantly
  - Leads to high-dimensional feature vectors

- Even when the counts are the same,

  the position and variance of the points in the cell can vary



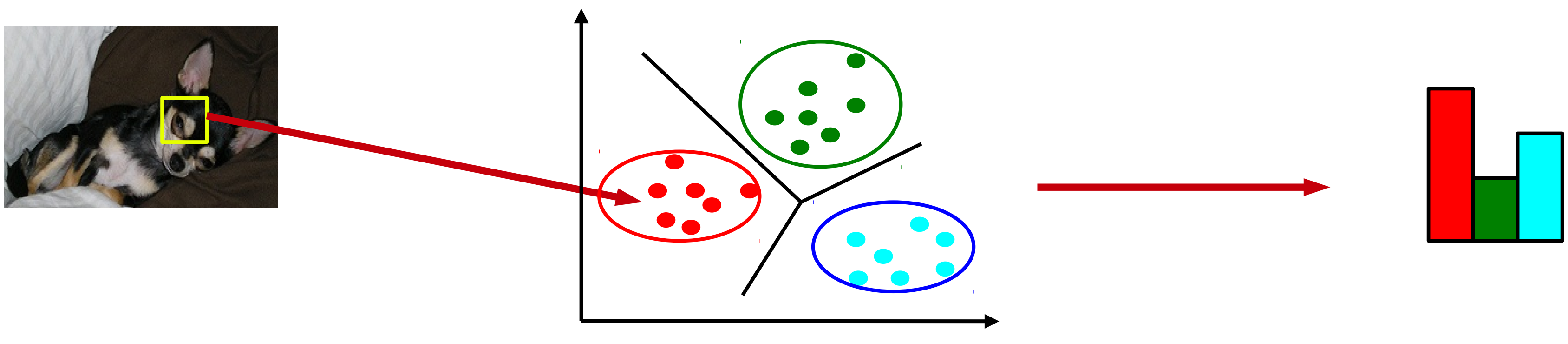
Application of FV for Gaussian mixture model of local features

- Gaussian mixture models for local image descriptors
  [Perronnin & Dance, CVPR 2007]
  - State-of-the-art feature pooling for image/video classification/retrieval

- Offline: Train k-component GMM on collection of local features

$$p(x)=\sum_{k=1}^{K}\pi_k N(x;\mu_k,\sigma_k)$$

- Each mixture component corresponds to a visual word
  - Parameters of each component: mean, variance, mixing weight
  - We use diagonal covariance matrix for simplicity
    - Coordinates assumed independent, locally per Gaussian



Application of FV for Gaussian mixture model of local features

- Model local image features with Gaussian mixture model

$$p(x) = \sum_{k=1}^{K} \pi_k N(x; \mu_k, \sigma_k)$$

- Fisher vector representation: gradient of log-likelihood
  - For the means and variances we have:

$$F^{-1/2} \nabla_{\mu_k} \ln p(x_{1:N}) = \frac{1}{\sqrt{\pi_k}} \sum_{n=1}^{N} p(k|x_n) \frac{(x_n - \mu_k)}{\sigma_k}$$

$$F^{-1/2} \nabla_{\sigma_k} \ln p(x_{1:N}) = \frac{1}{\sqrt{2\pi_k}} \sum_{n=1}^{N} p(k|x_n) \left\{ \frac{(x_n - \mu_k)^2}{\sigma_k^2} - 1 \right\}$$

  - Soft-assignments given by component posteriors

$$p(k|x_n) = \frac{\pi_k N(x_n; \mu_k, \sigma_k)}{p(x_n)}$$

Image representation using Fisher kernels

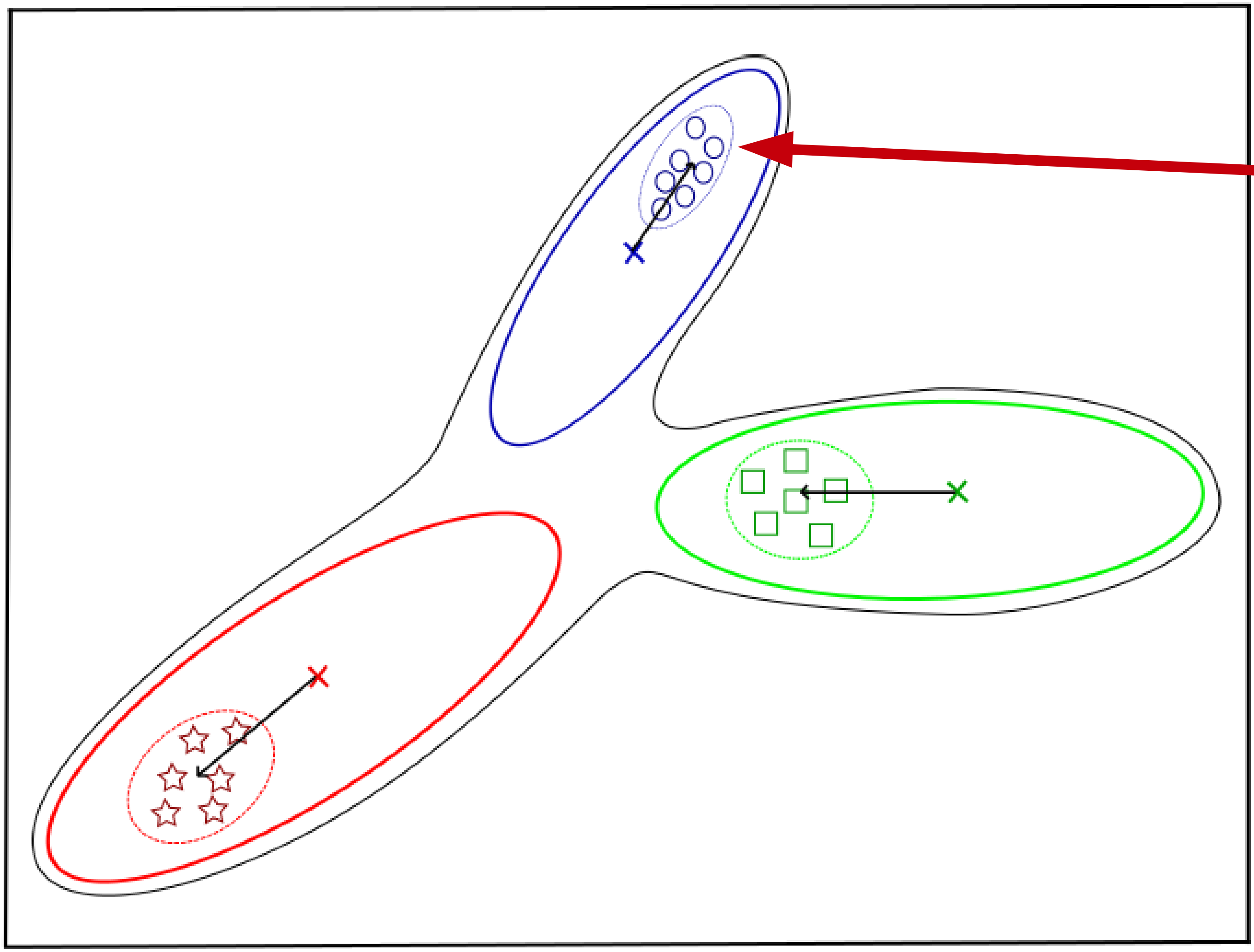
- Data representation

$$G(X,\Theta)=F^{-1/2}\left(\frac{\partial L}{\partial \alpha_1}, \; \ldots \; ,\frac{\partial L}{\partial \alpha_K} \;, \; \nabla_{\mu_1} L, \; \ldots \; ,\nabla_{\mu_K} L \;, \; \nabla_{\sigma_1} L, \; \ldots \;, \; \nabla_{\sigma_K} L \right)^T$$

- In total K(1+2D) dimensional representation, since for each visual word / Gaussian we have
  - Mixing weight (1 scalar)
  - Mean (D dimensions)
  - Variances (D dimensions, since single variance per dimension)

- Gradient with respect to mixing weights often dropped in practice since it adds little discriminative information for classification.
  - Results in 2KD dimensional image descriptor

Illustration of gradient w.r.t. means of Gaussians



New Data Points

BoW and FV from a function approximation viewpoint

- Let us consider uni-dimensional descriptors: vocabulary quantizes real line
- For both BoW and FV the representation of an image is obtained by sum-pooling the representations of descriptors.
  - Ensemble of descriptors sampled in an image $X=\{x_{1,} \dots, x_N\}$
  - Representation of single descriptor
    - One-of-k encoding for BoW $\phi(x_i)=[0,\dots,0,1,0,\dots,0]$
    - For FV concatenate per-visual word gradients of form

$$\phi(x_i)=\left(\dots, p(k|x_i)\left[1 \quad \frac{(x_i-\mu_k)}{\sigma_k} \quad \frac{(x_i-\mu_k)^2-\sigma_k^2}{\sigma_k^2}\right], \dots\right)$$

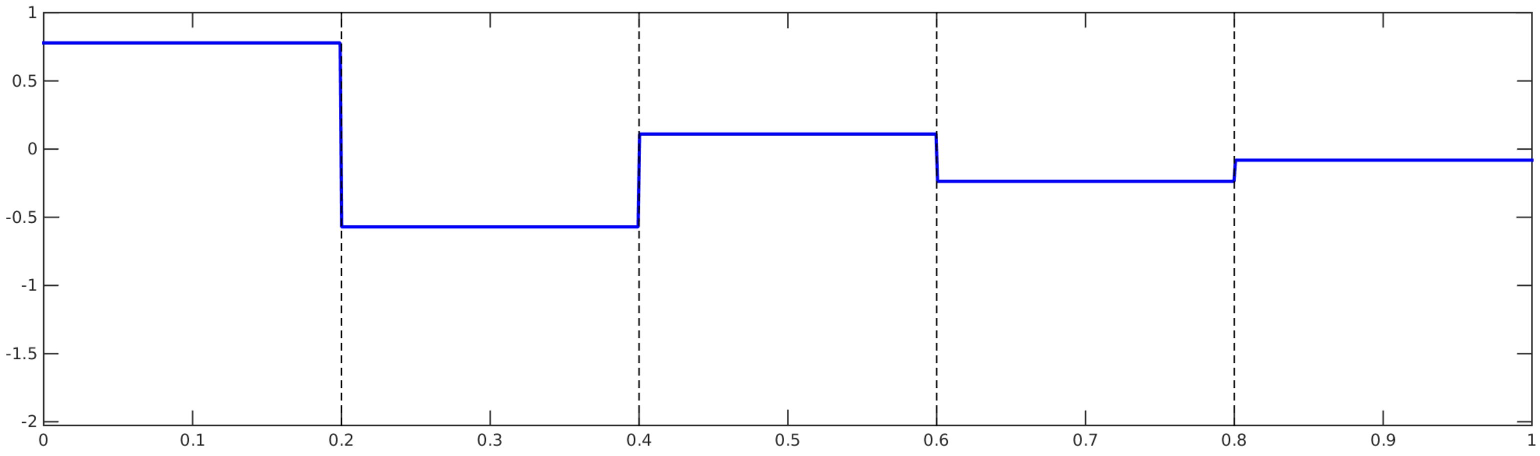
- Linear function of sum-pooled descriptor encodings is a sum of linear functions of individual descriptor encodings:

$$\Phi(X)=\sum_{i=1}^{N} \phi(x_i)$$

$$w^T \Phi(X)=\sum_{i=1}^{N} w^T \phi(x_i)$$

From a function approximation viewpoint

- Consider the score of a single descriptor for BoW
  - If assigned to k-th visual word then $w^T \phi(x_i) = w_k$
  - Thus: constant score for all descriptors assigned to a visual word



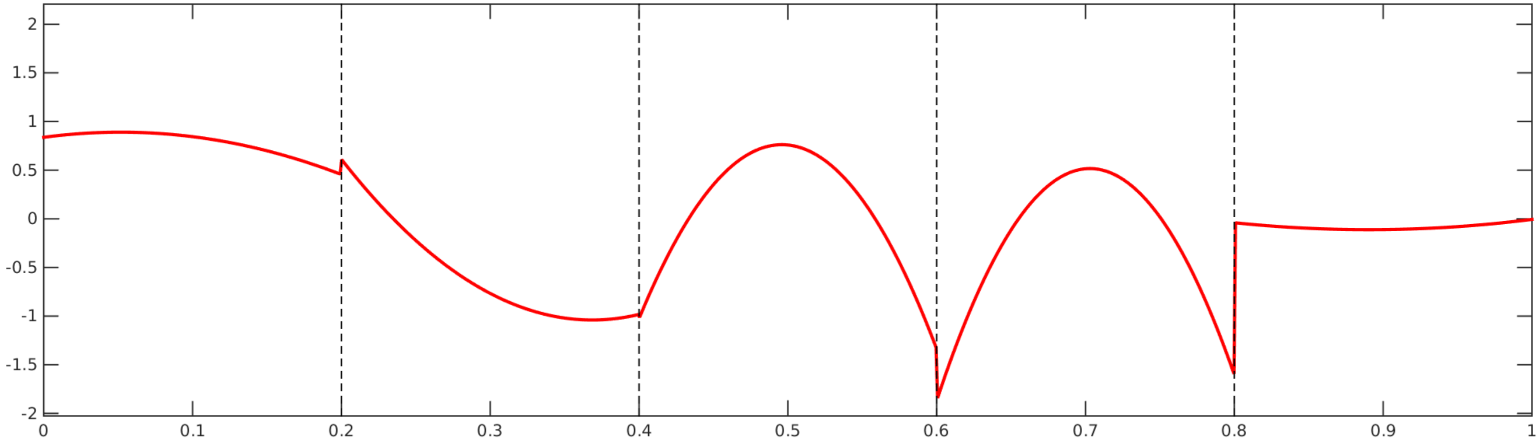
Each cell corresponds to a visual word

From a function approximation viewpoint

- Consider the same for FV, and assume soft-assignment is “hard”
  - Thus: assume for one value of k we have $p(k|x_i) \approx 1$
  - If assigned to the k-th visual word:

$$w^T \phi(x_i) = w_k^T \begin{bmatrix} 1 & \frac{(x_i - \mu_k)}{\sigma_k} & \frac{(x_i - \mu_k)^2 - \sigma_k^2}{\sigma_k^2} \end{bmatrix}$$

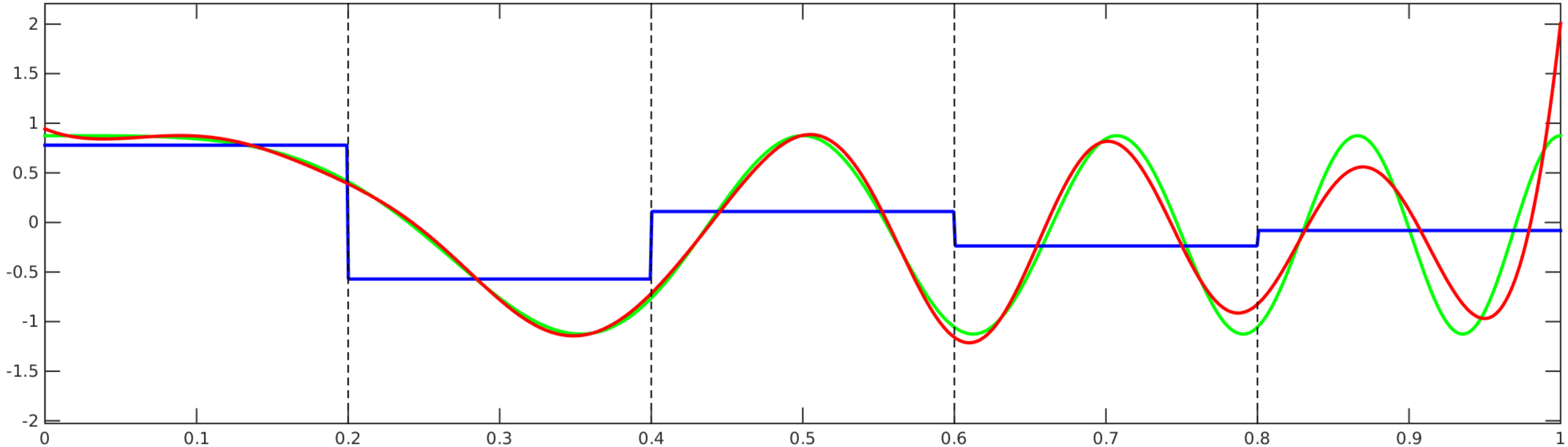
    - Note that w_k is no longer a scalar but a vector
  - Thus: score is a second-order polynomial of the descriptor x, for descriptors assigned to a given visual word.



From a function approximation viewpoint

- Consider that we want to approximate a true classification function (green) based on either BoW (blue) or FV (red) representation
  - Weights for BoW and FV representation fitted by least squares to optimally match the target function

- Better approximation with FV
  - Local second order approximation, instead of local zero-order
  - Smooth transition from one visual word to the next



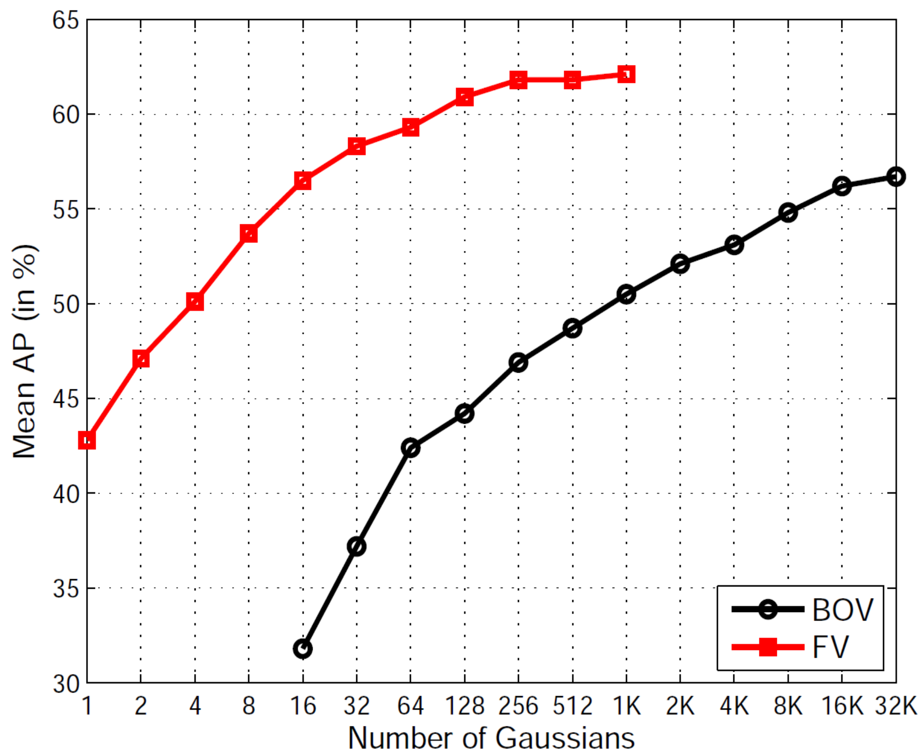
Fisher vectors: classification performance VOC'07

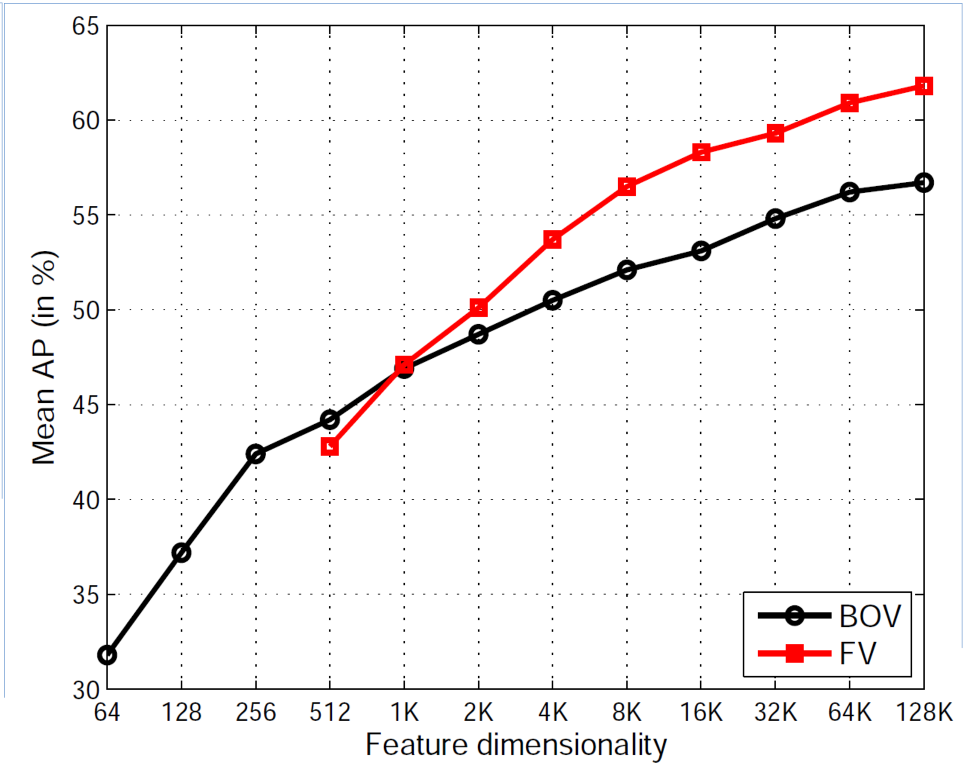
- Yearly evaluation from 2005 to 2012 for image classification

Fisher vectors: classification performance VOC'07

- Fisher vector representation yields better performance for a given number of Gaussians / visual words than Bag-of-words.

- For a fixed dimensionality Fisher vectors perform better, and are more efficient to compute

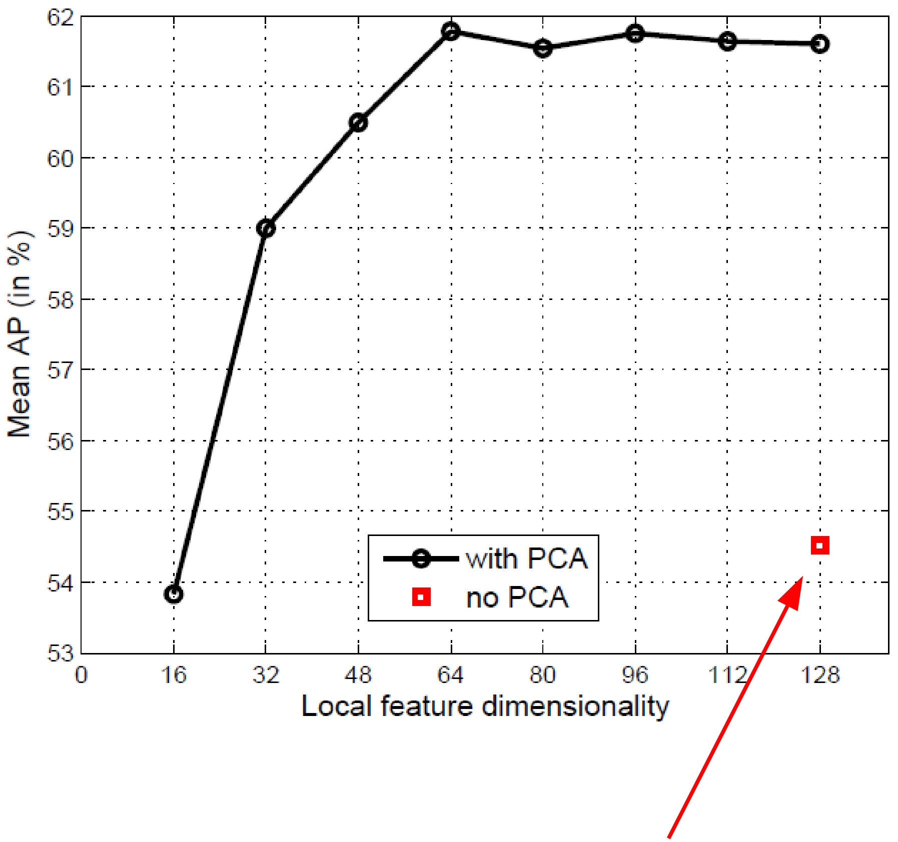




PCA dimension reduction of local descriptors

- We use diagonal covariance model
- Dimensions might be correlated
- Apply PCA projection to
  - De-correlate features
  - Reduce dimension of final FV
- FV with 256 Gaussians over local SIFT descriptors of dimension 128

Results on PASCAL VOC’07:



Normalization of the Fisher vector

- Inverse Fisher information matrix *F*
  - Renders FV invariant for re-parametrization
  - Linear projection, analytical approximation for MoG gives diagonal matrix
    [Sanchez, Perronnin, Mensink, Verbeek IJCV'13]

$$F = E\left[g(x)g(x)^T\right]$$
$$f(x) = F^{-1/2}g(x)$$

- Power-normalization, applied independently per dimension
  - Renders Fisher vector less sparse
    [Perronnin, Sanchez, Mensink, ECCV'10]
  - Corrects for poor independence assumption on local descriptors
    [Cinbis, Verbeek, Schmid, PAMI'15]

$$f(x) \leftarrow sign(f(x))|f(x)|^{\rho}$$
$$0 < \rho < 1$$

- L2-normalization
  - Makes representation invariant to number of local features
  - Among other Lp norms the most effective with linear classifier
    [Sanchez, Perronnin, Mensink, Verbeek IJCV'13]

$$f(x) \leftarrow \frac{f(x)}{\sqrt{f(x)^T f(x)}}$$

Effect of power and L2 normalization in practice

- Classification results on the PASCAL VOC 2007 benchmark dataset.
- Regular dense sampling of local SIFT descriptors in the image
  - PCA projected to 64 dimensions to de-correlate and compress
- Using mixture of 256 Gaussians over the SIFT descriptors
  - FV dimensionality: 2*64*256 = 32 * 1024

Power Nomalization	L2 normalization	Performance (mAP)	Improvement over baseline
No	No	51.5	0
Yes	No	59.8	8.3
No	Yes	57.3	5.8
Yes	Yes	61.8	10.3

Can you guess what is behind the masked area ?

- Obviously yes, since image regions are far from i.i.d.
- Yet Bag-of-word and GMM Fisher Vector representations assumes i.i.d. data

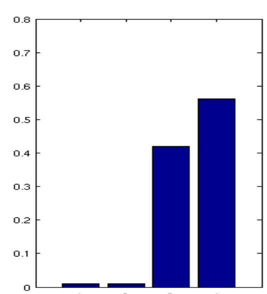
  [Cinbis, Verbeek, Schmid, PAMI 2015]

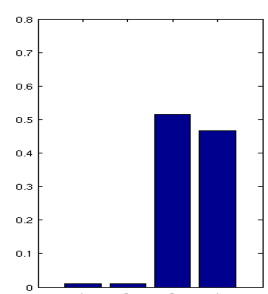
What's wrong with iid image representations ?

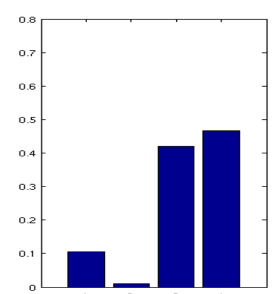
- Linear classification with BoW histograms: $f(h+\Delta)=w^T(h+\Delta)=w^T h+w^T\Delta$
  - Each occurrence of a visual word index leads to same score increment
  - Fisher vector over MoG: similar linear score change as in BoW model
  - Classification score proportional to object size !




- Retrieval
  - Distances of form d(x,y) = f(x-y) do not discount for small changes in large values: | 150 – 160 | = 10 = | 1 - 11 |
  - Dot product scoring is linear given the query image, just like the linear classifier case

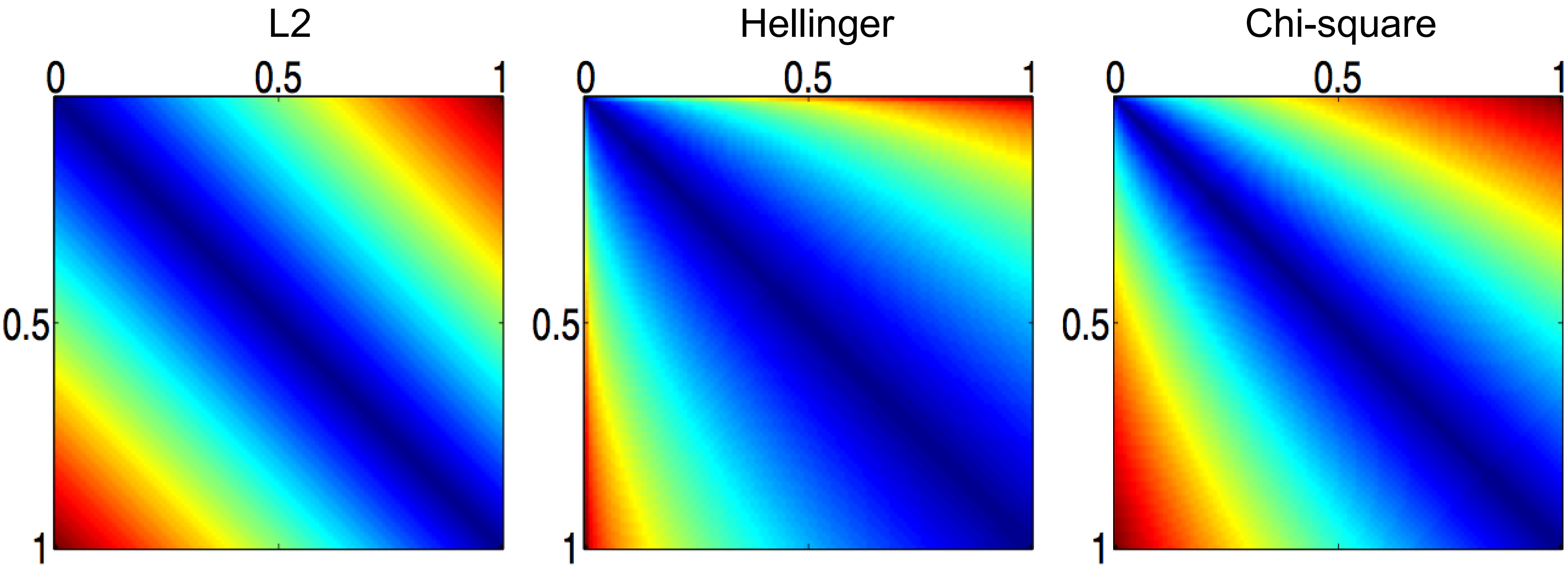




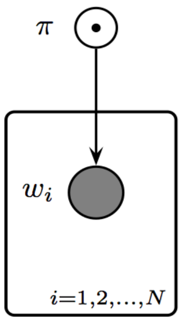


"Tricks" to improve BoW image representation

- Discounting of small changes in large values, limiting influence of burstiness
  - Chi-square distance between vectors $$d(x,y)=\frac{1}{2}\frac{(x-y)^2}{x+y}$$
  - Hellinger distance: element-wise square-rooting (Fisher Vector power normalization) $$d(x,y)=(\sqrt{x}-\sqrt{y})^2$$



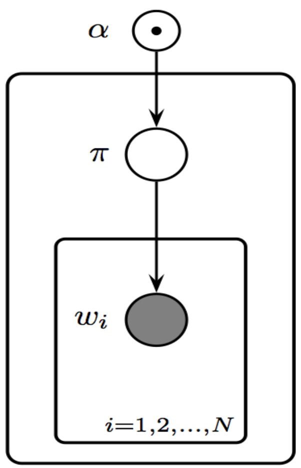
But how about Fisher vectors of non-iid models ?



- Standard BoW: Single universal multinomial governs all images
  - Sample patches iid from the universal multinomial model

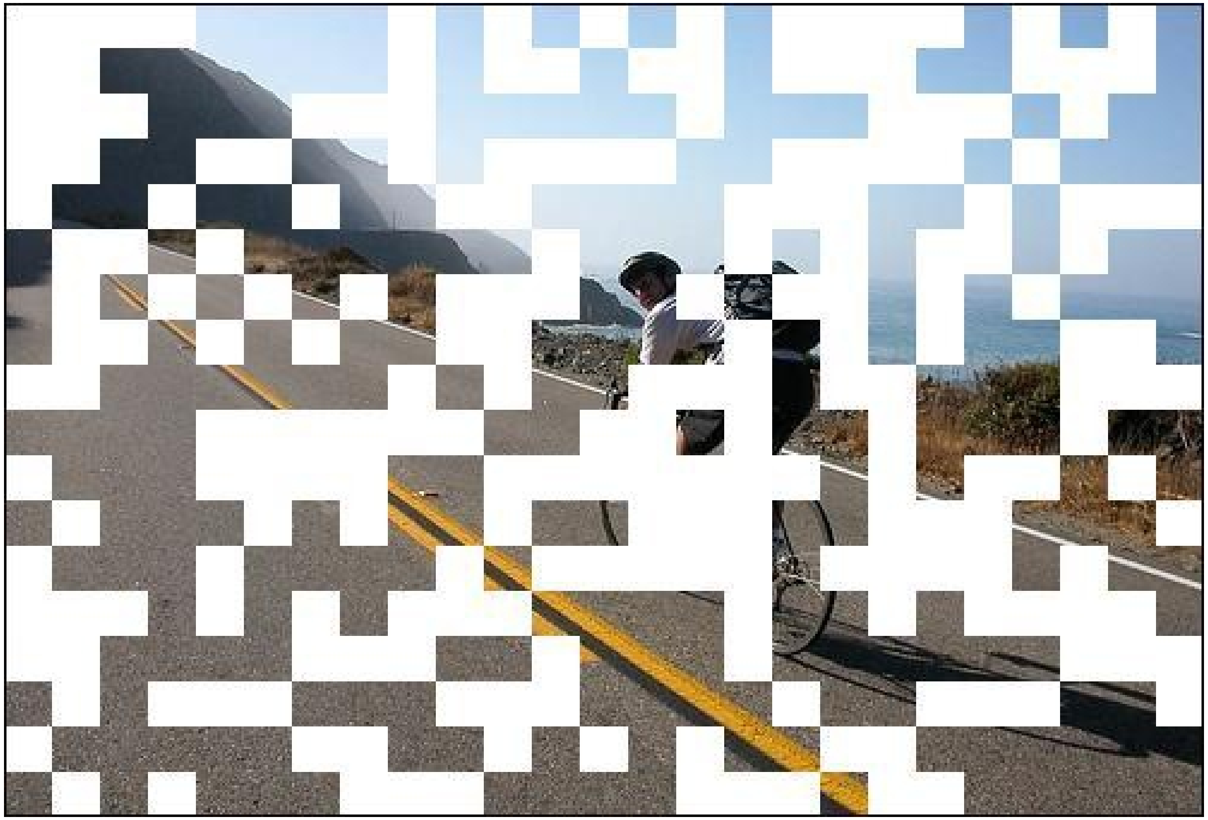
- **Compound Dirichlet–multinomial model** (a.k.a. Multivariate Pólya distribution) **assumes there is a *latent multinomial per image***
  - First, sample a multinomial image model from Dirichlet prior
  - Then, sample each word iid from multinomial image model
  - New hyper-parameter alpha

$$p(\pi) = Dir(\alpha)$$
$$p(w=k|\pi) = \pi_k$$
$$p(w_{1:n}) = \int p(\pi) \prod_{i=1}^{n} p(w_i|\pi)$$



  - **Latent multinomial generates full dependency across patches in an image**

Latent multinomial generates full dependency across patches



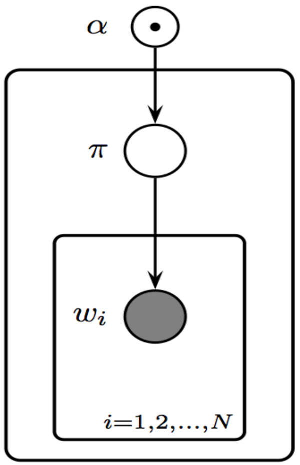
$$p(w_{n+1}|w_{1:n}) = \int p(\pi|w_{1:n})\, p(w_n|\pi)$$

- After we observe many patches of road, sky, bike, ….
- We infer that multinomial is likely to assign high likelihood to such patches
- Therefore, we expect to see even more such patches in the rest of the image

Fisher vector non-iid model

- Compound Dirichlet–multinomial model (a.k.a. Multivariate Pólya distribution)

$$p(w_{1:n}) = \int p(\pi) \prod_{i=1}^{n} p(w_i|\pi)$$
$$p(\pi) = Dir(\alpha)$$
$$p(w_i = k|\pi) = \pi_k$$

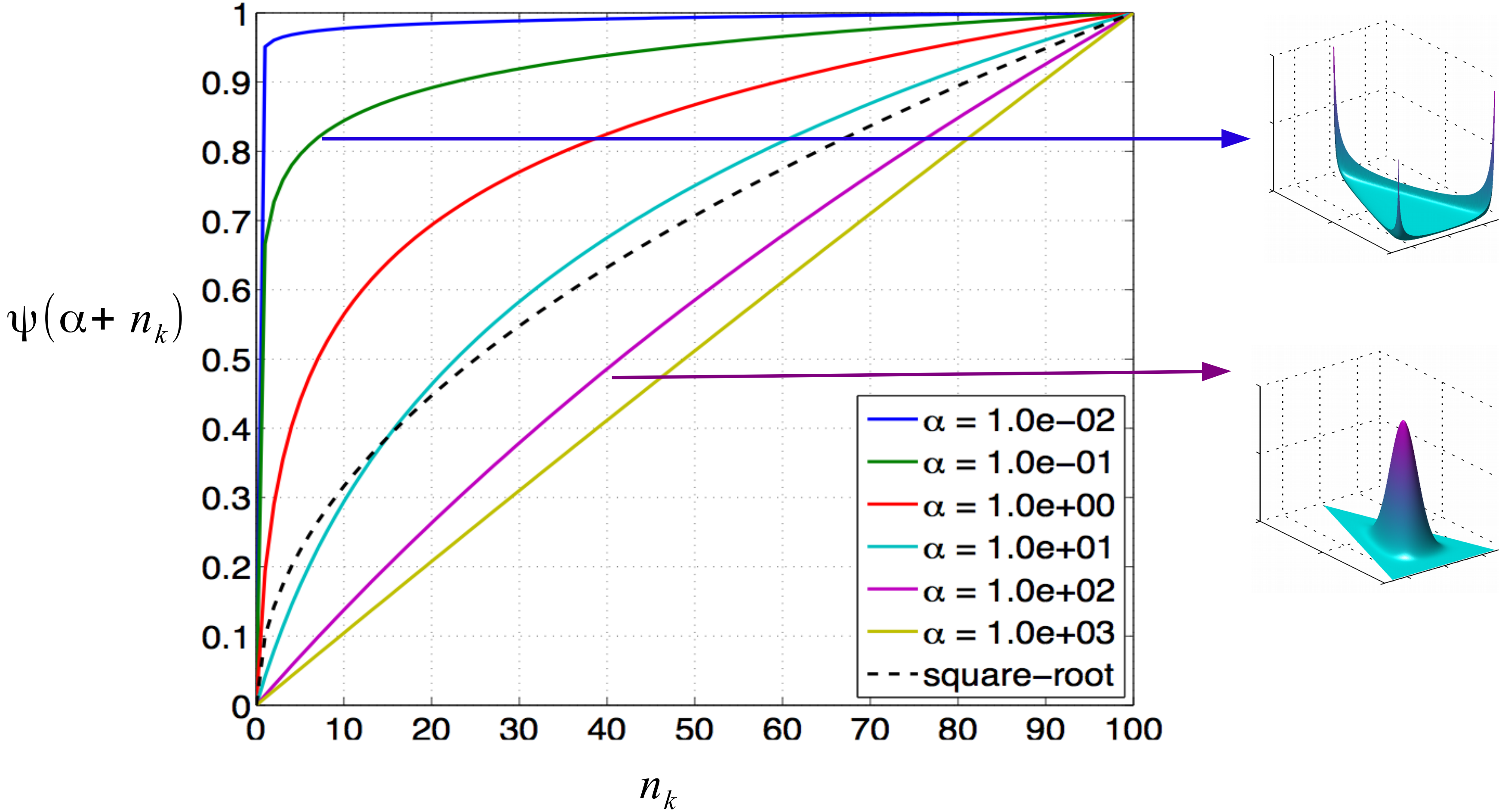


- Gradient given by di-gamma function of word counts $\mathbf{n_k}$ + parameter alpha

$$\frac{\partial \ln p(w_{1:n})}{\partial \alpha_k} = \psi(\alpha_k + n_k) + const.$$

Gradient: transformations on counts

- Small alpha > sparse Dirichlet prior > monotone concave, like sqrt
- Large alpha > dense Dirichlet prior > linear, like BoW histogram

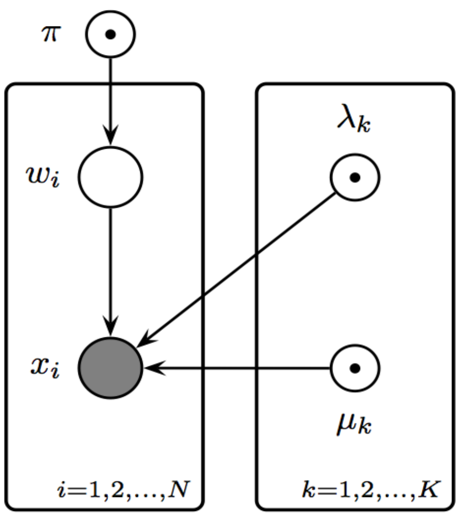


Fisher vector Gaussian mixture model

- Fisher vectors for Mixture of Gaussians (MoG) [Perronnin & Dance, CVPR'07]
  - Gaussian over feature space per visual word
  - Local (SIFT) descriptors are iid draws from “universal” MoG
  - State-of-the-art representation for image categorization (+sqrt transform)

$$p(x)=\sum_k p(w=k)\,p(x|w=k)=\sum_k \pi_k N(x;\mu_k,\Sigma_k)$$

$$p(x_{1:n})=\prod_i p(x_i)$$



- Gradient of log-likelihood of descriptors in image
  - High-dimensional image descriptor: K(2D+1)

$$\frac{\partial L}{\partial \alpha_k}=h_k-\pi_k$$

$$\frac{\partial L}{\partial \mu_k}=h_k\Sigma_k^{-1}(\mu_k-\tilde{\mu}_k) \qquad \tilde{\mu}_k=\frac{1}{n}\sum_i p(w=k|x_i)\,x_i$$

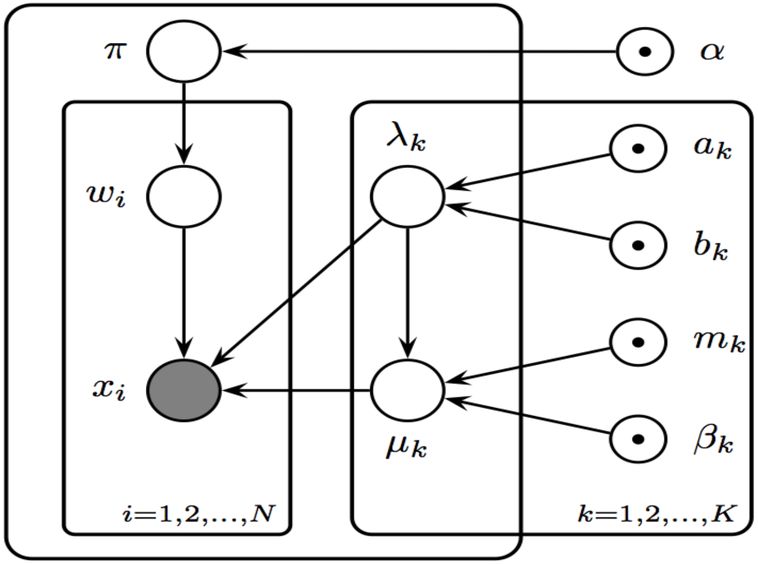
$$\frac{\partial L}{\partial \Sigma_k^{-1}}=h_k\frac{1}{2}(\Sigma_k-\tilde{\Sigma}_k)$$

Latent mixture of Gaussian (MoG) model

- To remove iid assumption we proceed as before:
  - Treat image-specific MoG model as latent variable
  - Put priors on: mixing weights, variances, and means:

$$p(\pi) = Dir(\pi|\alpha)$$
$$p(\lambda) = Gam(\lambda|a, b)$$
$$p(\mu|\lambda) = N(\mu|\mu_{0,}(\beta\lambda)^{-1})$$

- Generative process per image
  - Sample MoG parameters from prior distributions
  - Sample descriptors iid from image-specific MoG



$$p(x_{1:n}) = \int p(\pi, \mu, \lambda) \prod_{i=1}^{n} p(x_i|\pi, \mu, \lambda)$$
$$p(x_i|\pi, \mu, \lambda) = \sum_k \pi_k N(x_i|\mu_k, \lambda_k^{-1})$$

Latent mixture of Gaussian model

- For this model computation of likelihood and its gradient are intractable
- Learning is done using a **Variational EM algorithm**
  - based on optimizing variational free-energy bound on the log-likelihood

$$V = \log p(x_{1:n}) - D_{KL}\left(q(w_{1:n}, \pi, \lambda, \mu) \| p(w_{1:n}, \pi, \lambda, \mu | x_{1:n})\right)$$
$$= H(q) + E_q\left[\log p(x_{1:n}, w_{1:n}, \pi, \lambda, \mu)\right]$$

- Constraining distribution **q** to have a certain independence structure both steps of EM algorithm become tractable

$$q(w_{1:n}, \pi, \lambda, \mu) = q(w_{1:n})\, q(\pi, \lambda, \mu)$$

- Use the gradient of the bound as an approximate Fisher Vector
  - In general, if bound is tight, then the exact Fisher vector is recovered
  - Generates similar discounting effects as observed for latent BoW model
    - Eg, for mixing weights same di-gamma function, now applied to soft-counts

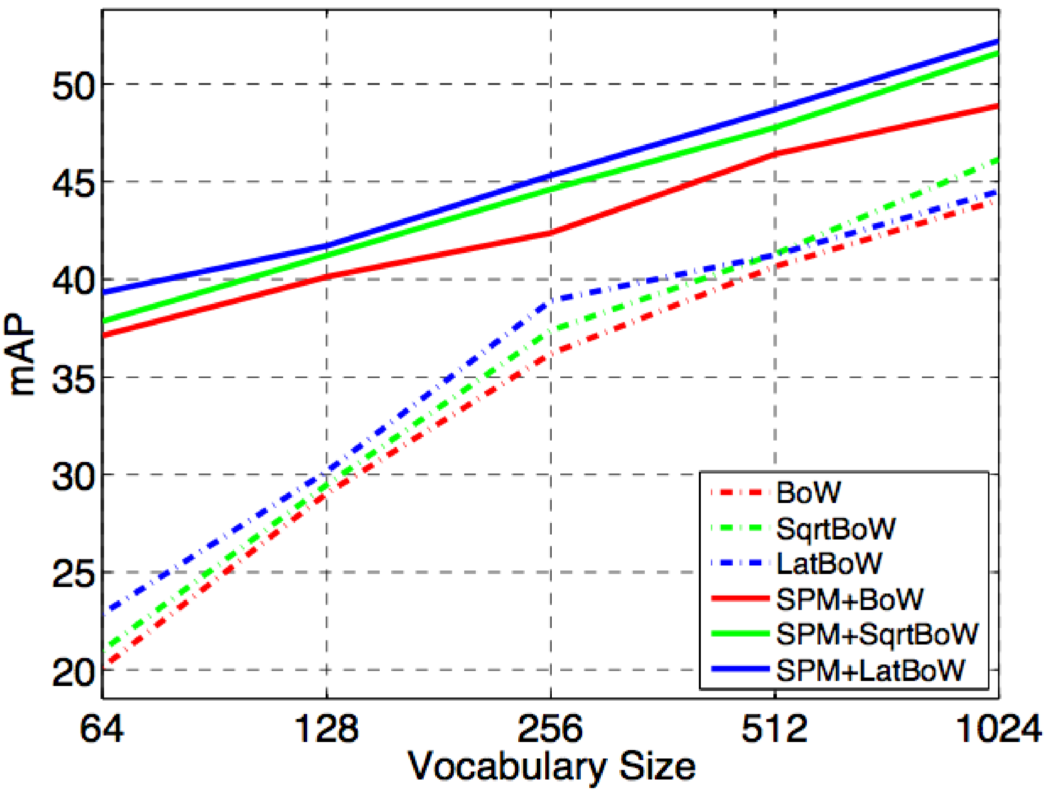
Experimental evaluation on image categorization task

- Data set: PASCAL VOC 2007
  - Images labeled for presence of 20 object categories
    - Airplane, bicycle, boat, bus, car, cat, cow, dog, horse, motorbike, person, …
  - 5000 images to train models, and 5000 images used for evaluation
- Performance measured in mean Average Precision over the 20 classes
- SIFT descriptors computed over dense multi-scale grid, PCA to 80 dim
- To incorporate spatial layout image representations computed over
  - Complete image, 4 quadrants, 3 horizontal bands

Evaluation Bag-of-word models

- Comparing linear classifiers based on
  - BoW histogram, sqrt of BoW histogram, latent BoW model Fisher Vector
  - Varying vocabulary size, and use of spatial pyramid (SPM)
- Latent BoW model and sqrt tansform lead to comparable improvement

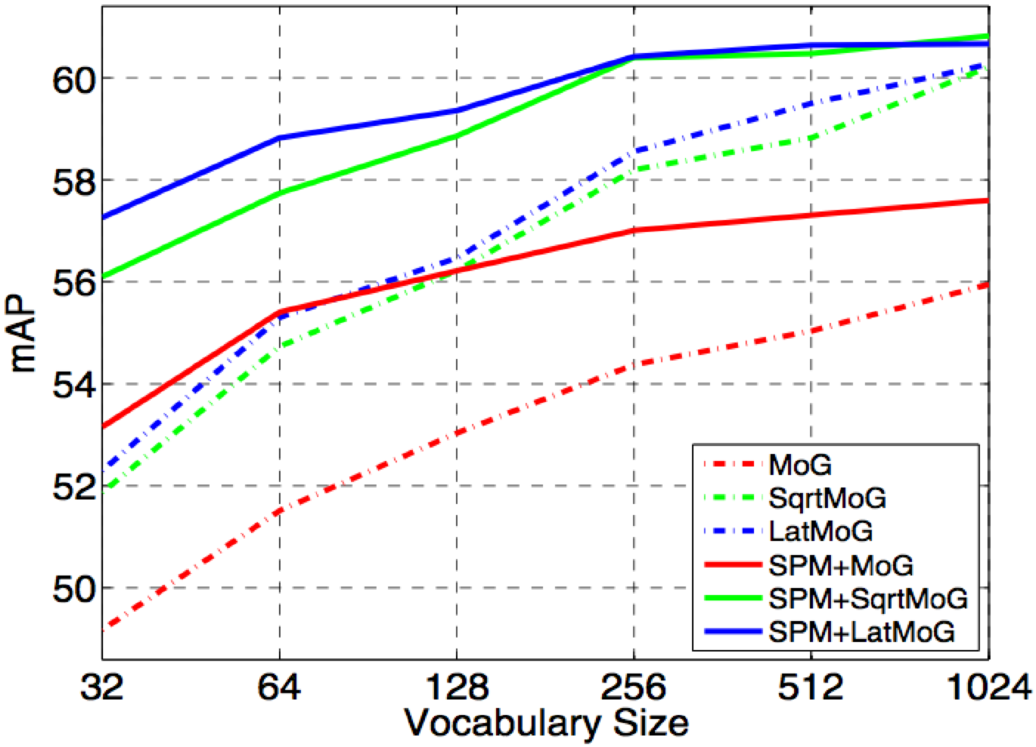
SPM	Method	64	128	256	512	1024
No	BoW	20.1	29.0	36.2	40.7	44.1
No	SqrtBoW	21.0	29.5	37.4	**41.3**	**46.1**
No	LatBoW	**22.9**	**30.1**	**38.9**	41.2	44.5
Yes	BoW	37.1	40.1	42.4	46.4	48.9
Yes	SqrtBoW	37.8	41.2	44.6	47.8	51.6
Yes	LatBoW	**39.3**	**41.7**	**45.3**	**48.7**	**52.2**



Evaluation Latent mixture of Gaussians model

- Comparing linear classifiers based on
  - Fisher Vector of MoG model, sqrt of MoG FV, Latent MoG model FV
  - Varying vocabulary size, and use of spatial pyramid (SPM)
- Latent MoG model and sqrt transform lead to comparable improvement
- Non-iid models explain effectiveness of FV power normalization
  - But computationally power normalization is much cheaper

SPM	Method	32	64	128	256	512	1024
No	MoG	49.2	51.5	53.0	54.4	55.0	55.9
No	SqrtMoG	51.9	54.7	56.2	58.2	58.8	60.2
No	LatMoG	**52.3**	**55.3**	**56.5**	**58.6**	**59.5**	**60.3**
Yes	MoG	53.2	55.4	56.2	57.0	57.3	57.6
Yes	SqrtMoG	56.1	57.7	58.9	**60.4**	60.5	**60.8**
Yes	LatMoG	**57.3**	**58.8**	**59.4**	**60.4**	**60.6**	60.7



Example applications: Fine-grained classification

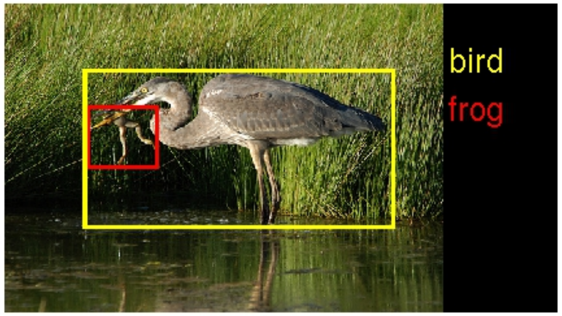
aircraft (100) birds (83) cars (196) dogs (120) shoes (70)

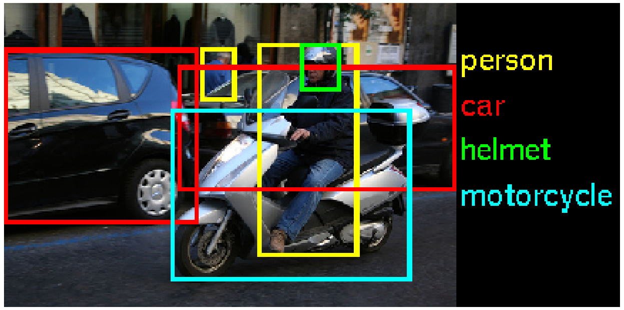
- Winning INRIA+Xerox system at FGComp'13:
  http://sites.google.com/site/fgcomp2013
  - multiple low-level descriptors: SIFT, color, etc.
  - Fisher Vector embedding

    [Gosselin, Murray, Jégou, Perronnin, "Revisiting the Fisher vector for fine-grained classification", PRL'14.]

- Many other successful uses of FVs for fine-grained recognition
  - Rodriguez and Larlus, "Predicting an object location using a global image representation", ICCV'13.
  - Gavves, Fernando, Snoek, Smeulders, Tuytelaars, "Fine-Grained Categorization by Alignments", ICCV'13
  - Murray, Perronnin, "Generalized Max Pooling", CVPR'14.

Example applications: object localization





- ImageNet'13 detection: http://www.image-net.org/challenges/LSVRC/2013/
- Winning system by University of Amsterdam
  - region proposals with selective search
  - Fisher Vector embedding
  - Fast Local Area Independent Representation (FLAIR)

Van de Sande, Snoek, Smeulders, "Fisher and VLAD with FLAIR", CVPR'14.

Example applications: face verification

- Face track description:
  - track face
  - extract SIFT descriptors
  - encode using Fisher vectors
  - pool at face track level

Parkhi, Simonyan, Veldaldi, Zisserman, "A compact and discriminative face track descriptor", CVPR'14.

- New state-of-the-art results on the YouTube faces dataset

	Method	Accuracy	AUC	EER
1	MGBS & SVM- [37]	78.9 ± 1.9	86.9	21.2
2	APEM FUSION [20]	79.1 ± 1.5	86.6	21.4
3	STFRD & PMML [11]	79.5 ± 2.5	88.6	19.9
4	VSOF & OSS (Adaboost) [22]	79.7 ± 1.8	89.4	20.0
5	Our VF^2 (restricted)	83.5 ± 2.3	92.0	16.1
6	Our VF^2 (restricted & flip)	84.7 ± 1.4	93.0	14.9
7	Our VF^2 (unrestricted & flip)	83.5 ± 2.1	94.0	13.0
8	Our VF^2 (unrestricted & jitt. pool.)	83.8 ± 1.6	95.0	**12.3**

Example: action recognition and localization

- THUMOS action recognition challenge 2013 & 2014

  http://crcv.ucf.edu/ICCV13-Action-Workshop

- Winning systems by INRIA-LEAR
  - improved dense trajectory video features
  - Fisher Vector embedding

Wang, Oneata, Verbeek and Schmid, "A robust and efficient video representation for action recognition", IJCV'15.