Variable Bandwidth QMDPE and Its Application in Robust Optical Flow Estimation

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Abstract

Robust estimators, such as Least Median of Squared (LMedS) Residuals, M-estimators, the Least Trimmed Squares (LTS) etc., have been employed to estimate optical flow from image sequences in recent years. However, these robust estimators have a breakdown point of no more than 50%. In this paper, we propose a novel robust estimator, called variable bandwidth Quick Maximum Density Power Estimator (vbQMDPE), which can tolerate more than 50% outliers. We apply the novel proposed estimator to robust optical flow estimation. Our method yields better results than most other recently proposed methods, and it has the potential to better handle multiple motion effects.

1. Introduction

One major task of computer vision is to compute the optical flow from image sequences [2, 3, 6, 10, 11, 13, 15, 17, 19, 20, 22, 26]. Accurate computation of optical flow is an important foundation for tasks, such as motion segmentation, extracting structure from motion, etc. Traditional methods of computing optical flow are non-robust. Which means they will fail to correctly compute optical flow when the two assumptions: data conservation and spatial coherence, are violated. Clearly, these assumptions will be violated near motion boundaries, and when shadows, occlusions, and/or transparent motions are present.

During the last ten years, robust techniques, such as: M-estimators, Least Median Squares (LMedS) estimator, Least Trimmed Squares (LTS) estimators, and robust Total Least Squares (TLS) estimator, etc., have been employed to extract optical flow [1, 4, 5, 24, 25, 35]. Because these robust estimators can tolerate the influence of “bad” data, i.e. outliers, they usually obtain better results. Unfortunately, these robust estimators have a breakdown point no more than 50%. This means that when the data contain more than 50% outliers, these estimators will totally breakdown. Such will happen, for example, near motion boundaries involving more than two different motions.

In this paper, we will provide, based on our previous work [33, 34], a novel robust estimator—variable bandwidth QMDPE. Instead of using a fixed bandwidth as in QMDPE, vbQMDPE uses data-driven bandwidth selection. We apply the novel proposed robust estimator to the task of optical flow computation. We also correct the results of Bab-Hadiashar and Suter [1] for the Otte image sequence.

vbQMDPE seems to rarely breakdown if the percentage of outliers is less than 80%, outperforming most other methods in optical flow computation. Of course, any method can breakdown under extreme data: even LMedS and LTS can breakdown when clustered outliers are present - despite those outliers constituting less than 50% of the whole data [33].

2. Optical Flow Computation

Let I(x, y, t) be the luminance of a pixel at position (x, y) and time t, and **v** = (u, v) be the optical flow. The data conservation assumption implies [12]:

$$I(x, y, t) = I(x + u\delta t, y + v\delta t, t + \delta t) \qquad (1)$$

First order expansion yields the optical flow constraint (OFC) equation:

$$\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \frac{\partial I}{\partial t} = 0 \qquad (2)$$

where (∂I/∂x, ∂I/∂y, and ∂I/∂t) are partial derivatives of luminance I w.r.t. space and time at point (x, y, t).

The residual at (x, y) can be written as:

$$r(u,v) = \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \frac{\partial I}{\partial t} \qquad (3)$$

The error measure using the least squares (LS) within the small local neighborhood R can be written as:

$$E_{LS}(u,v) = \sum_{(x,y)\in R} |r(u,v)|^2 \qquad (4)$$

From equation (2), we can see there is only one equation but with two variables to estimate - the aperture problem. In order to constrain the solution, the local region R should be as large as possible. However, if R is too large, the spatial coherence assumption will be violated - the generalized aperture problem [5]. The affine motion model of image flow is sometimes used in preference to the constant flow model:

$$u = a_0 + a_1 x + a_2 y$$
$$v = a_3 + a_4 x + a_5 y \qquad (5)$$

Traditional (Least Squares) methods estimate the optical flow by minimizing the error measure in equation (4), assuming a flow model such as (5).

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3. Robust Statistics and vbQMDPE

It is almost unavoidable that data are contaminated (due to occlusion, shadow, transparent motion, faulty feature extraction, sensor noise, etc) and it is also likely that the data will include multiple structures (such as multiple motion). Although the least squares (LS) estimator is high efficient when the data have a Gaussian distribution, it is extremely sensitive to outliers. The breakdown point of an estimator may be roughly defined as the smallest percentage of outlier contamination that can cause the estimator to produce arbitrarily large values [29]. The LS estimator has a breakdown point of 0%. The maximum-likelihood-type estimators (M-estimators) [16] can reduce the influence of outliers, yet they have breakdown points less than 1/(p+1), where p is the number of the parameters to estimate. Rousseeuw proposed the LMedS and the LTS estimators [28, 29] that can tolerate up to 50% outliers.
More recently, some new promising robust estimators, which can tolerate more than 50% outliers, appeared [18, 21, 31, 36]. However, RESC [36] needs the user to tune many parameters, MUSE [21] needs a lookup table for the scale estimator correction and ALKS[18] is limited in its ability to handle extreme outliers and its lack of stability under a small percentage of outliers [33].
In the next subsection, we will, based on our previous work [33, 34], provide a novel *variable bandwidth* QMDP estimator.

3.1 Variable bandwidth QMDPE

Our starting point is based on two assumptions:

(1) The inliers have a Gaussian-like distribution;
(2) The inliers are a relative majority of the data.

The first assumption is a common one. However, traditional robust estimators (LMedS, LTS, etc.), assume the inliers should occupy an absolute majority of the whole data – violating (2).
A crucial part of our algorithm is the use of the mean-shift procedure[8, 14] - we employ it to find the position of the local maximum density in signed residual space. The convergence of the mean shift iterations can be found in [8]. We run the mean shift iterations with initial position of zero in residual space and with bandwidth h, then we obtained the center of the converged window Xc.
When a model is correctly fitted: (1) The center of the converged window (Xc) in residual space should be as close to zero as possible; and (2) The probability density $\hat{f}$ (X_c) of the point at Xc should be as high as possible. So we define our objective function ψ_{DP} as:

$$\psi_{DP} = \frac{\left(\hat{f}(Xc)\right)^{\alpha}}{\exp(|Xc|)} \qquad (6)$$

where α is a factor that adjusts the relative influence of the probability density to the residual of the point corresponding to the center of the converged window (experimentally set to 2.0).
The probability density can be estimated by:

$$\hat{f}(x) = \frac{1}{nh^d}\sum_{i=1}^{n} K\left(\frac{x - X_i}{h}\right) \qquad (7)$$

where d is the dimension of Euclidean space; n is the number of the data points, h the band-width.
The Epanechnikov kernel [30] K is employed:

$$K_e(x) = \begin{cases} \frac{1}{2}c_d^{-1}(d+2)(1-x^T x) & if\ x^T x < 1 \\ 0 & otherwise \end{cases} \qquad (8)$$

where c_d is the volume of the unit d-dimensional sphere, e.g., $c_1=2$, $c_2=\pi$, $c_3=4\pi/3$.
One crucial issue in the non-parametric density estimation, and the mean shift method, is how to choose h [8, 9, 32]. We employ a method from [32]:

$$\hat{h} = \left[\frac{243R(K)}{35u_2(K)^2 n}\right]^{1/5} s \qquad (9)$$

where $R(K) = \int_{-1}^{1} K(\zeta)^2 d\zeta$ and $u_2(K) = \int_{-1}^{1} \zeta^2 K(\zeta) d\zeta$, s is the sample standard derivation. A robust median scale estimator is then given by [29]:

$$s = 1.4826\,\mathrm{med}_i x_i \qquad (10)$$

$\hat{h}$ will provide an above bound on the AMISE (asymptotic mean integrate error) optimal bandwidth $\hat{h}_{AMISE}$, thus we choose the bandwidth as c$\hat{h}$, c is a constant number (0<c<1) and is used to avoid over-smoothing.

3.2 The algorithm: variable bandwidth QMDPE

The vbQMDPE procedure is:

(1) Randomly choose one p-subset, estimate the model parameters by the p-subset, and calculate the residuals of all data points.
(2) Adaptively choose bandwidth h using the method described in sec. 3.1.
(3) Apply the mean shift iteration in the residual space with initial window center zero. Thus, we obtain the center of converged window *Xc*.
(4) Calculate the probability density $\hat{f}$ (X_c) at the position *Xc* by equation (7) and (8).
(5) Calculate the density power according to equation (6).
(6) Repeat step (1) to step (5) m times. Finally, output the parameters with maximum density power.

Enough random subsets m are used for a high probability P that at least one “clean” p-subset is chosen:

$$m = \frac{\log(1-P)}{\log[1-(1-\varepsilon)^p]} \qquad (11)$$

where ε is the fraction of outliers contained in the whole set of points.
“Variable bandwidth” means that the bandwidth is variable for each randomly chosen p-subset - instead of using a fixed bandwidth as in our previous work [33].

In order to improve the statistical efficiency, a weighted least square procedure [29] can be carried out.

We note that although Chen and Meer [7] also employ a kernel density estimation technique in parametric model estimation, they considered their mode of the density estimate in the projection space along the direction of parameter vector. Our method considers the density distribution of the mode in the residual space. The criteria of the two methods also differ.

We are aware that the median scale estimator in equation (9) may be biased for nonsymmetrical multi-model data; also, too small bandwidth without downward bound will introduce artifacts.

3.3 Performance of vbQMDPE.

We demonstrate vbQMDPE is very robust to outliers by comparing it to several other traditional methods (the LS, LMedS and LTS methods). We do not compare the proposed method to its predecessor (QMDPE) because the only essential difference is that now the bandwidth h is data driven.

In our first experiments, we take a simple setting — line fitting. We generated four kinds of data (one step, two steps, two crossed lines, and four lines), each with a total of 500 data points. The signals were corrupted by Gaussian noise with zero mean and unit standard variance. Among the 500 data points, α data points were randomly distributed in the range of (0, 100). The i'th structure has γ_i data points.

The four signals are as follows:

(a) One step: x:(0-55), y=30, γ_1=225; x:(55-100), y=40, γ_2=225; α=50.

(b) Two steps: x:(0-30), y=20, γ_1=100; x:(30-55), y=40, γ_2=100; x:(55-80), y=60, γ_3=100; α=200.

(c) Two crossed lines: x:(20-70), y=x+10, γ_1=150; x:(35-85), y=115-x, γ_2=150; α=200.

(d) Four lines: x:(0-25), y=3x+10, γ_1=75; x:(25-55), y=130-2x, γ_2=20; x:(40-65), y=3x-110, γ_3=75; x:(65-90), y=280-3x, γ_4=75; α=370.

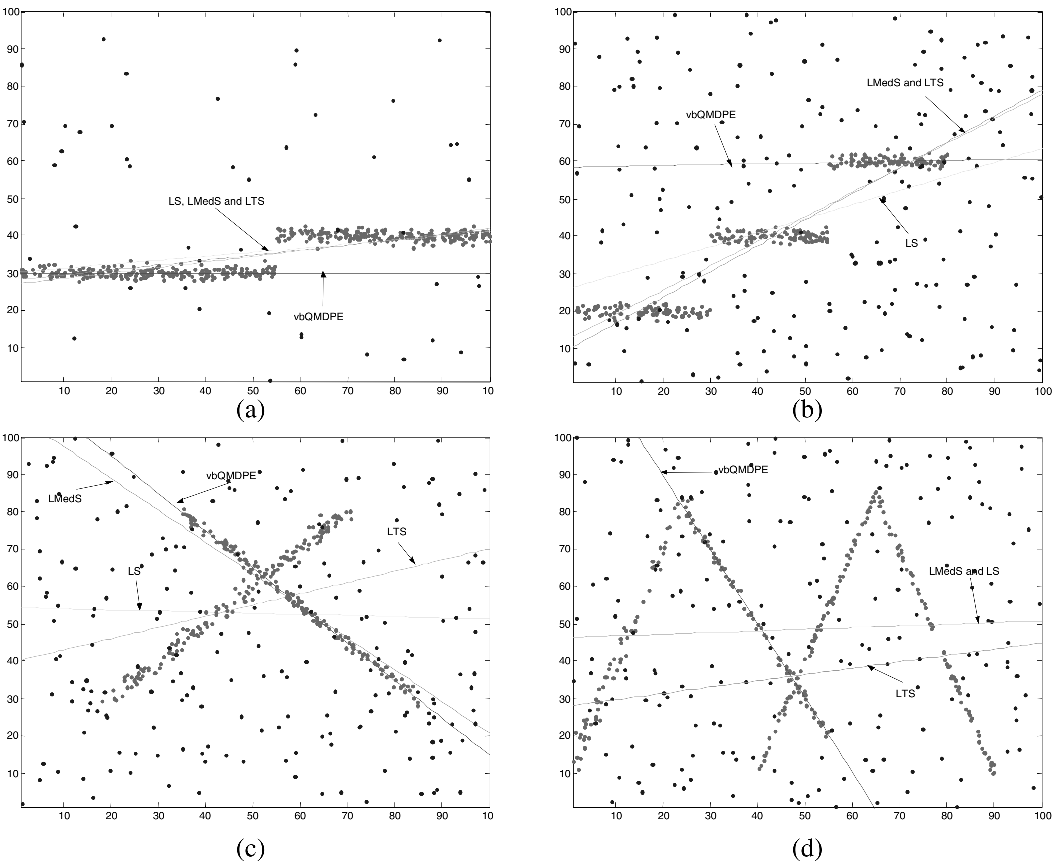


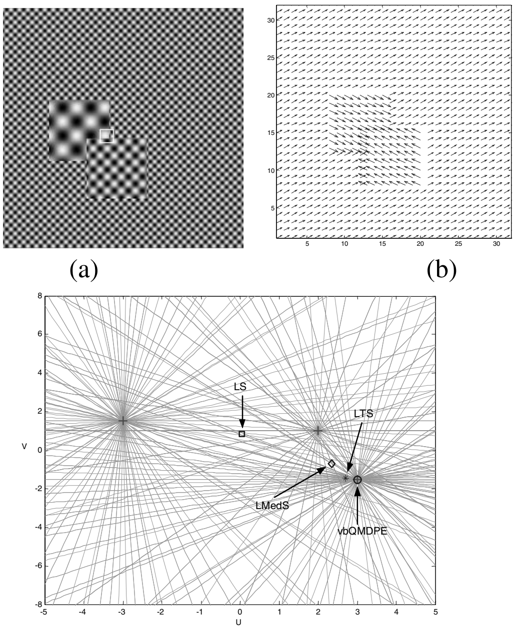
Figure 1. Comparing the performance of vbQMDPE, LS, LMedS, and LTS with (a) 55%; (b) 80%; (c) 70%; (d) 85% outliers.

From figure 1, we can see that LS is non-robust, and that LMedS and LTS failed. Only vbQMDPE correctly fitted all the four signals - not even breaking down when the data includes 85% outliers (figure 1 (d)).

4. vbQMDPE and optical flow

The optical flow constraint (OFC) is a linear equation in u-v space. Each pixel gives rise to one such linear constraint and, in a noise-free setting, and assuming constant u and v, all lines intersect at a common point.

Two main difficulties in optical flow estimation are [24]: (a) the discontinuities in the local velocity; and (b) the "aperture" problem. The first difficulty is related to occlusions between image illumination discontinuities, moving objects, or moving object boundaries. One solution to the second difficulty is to enlarge the local window so as to collect more constraint equations to over determine the optical; this will bring higher statistical efficiency. However, enlarging the window means more chance of including multiple motions (forming multiple clusters of intersecting lines e.g., figure 2). Because traditional estimators (M-estimators, LMedS, LTS, etc.) have only up to 50% breakdown point, they may fail to compute optical flow when the data include multiple motion structures (i.e. the outliers occupy more than 50% of the data). In such cases, vbQMDPE performs well.



(a) (b)

(c)

Figure 2. One example of multiple motions.

We generated a two-square image sequence using the method similar to that in [2]. Figure 2 (a) shows one snapshot of the image sequence. The correct optical flow is shown in figure 2 (b). The small window centered at (110, 136) in figure 2 (a) includes three motions: each motion involves less than 50% data points. Its OFC plot, using symbolically determined derivatives of I, is shown in figure 2 (c). From figure 2 (c), we can see that there are three motions included in the small window in figure 2 (a). The optical flow of each motion (2.0, 1.0), (-3.0, 1.5), (3.0, -1.5) is marked by red plus sign. The proposed robust estimator gives correct optical flow estimation (3.0, -1.5). However, by the LMedS method, the estimated optical flow is (2.36, -0.71); and the estimated optical flow by the least trimmed squares and the least squares method is respectively (2.71, -1.43) and (0.06, 0.84).

4.1 Variable-Bandwidth-QMDPE optical flow computation

The first step to compute optical flow is to estimate the spatio-temporal derivatives of the image brightness. We follow Bab-Hadiashar and Suter [1], and Nagel [23], by convolving the image brightness with derivatives of 3D spatio-temporal Gaussian function:

$$G(\mathrm{x}) = \frac{1}{(2\pi)^{3/2}\sqrt{|\Sigma|}} e^{-\frac{1}{2}\mathrm{x}^T\Sigma^{-1}\mathrm{x}} \qquad (13)$$

where $\mathbf{x} = (x, y, t)^T$; Σ is covariance matrix.

There are methods to estimate the derivatives *near the discontinuities of optical flow* (e.g., [35]). In our simple approach, we first estimate the derivatives of I with initial standard variance σ_0 then, when the estimated derivatives (I_x, I_y, and I_t) are larger than a threshold, we simply re-compute the derivatives with half of the standard variance in that corresponding direction.

For each NxN patch of the image and chosen motion model (in our case, constant motion model and affine motion model), we solve for the flow using vbQMDPE.

The measure of reliability in [1] can be employed in our method.

4.2 Quantitative error measures for optical flow

When the "ground truth" optical flow of image sequences is known, the error analysis is performed by Barron's method [2]. The angular error measures is reported in degree:

$$\mathrm{E} = \arccos(\mathbf{v_e}, \mathbf{v_c}) \qquad (14)$$

where $\mathbf{v_e} = (u, v, 1)^T / \sqrt{u^2 + v^2 + 1}$ and $\mathbf{v_c}$ is the true motion vector. The average and standard deviation of the errors are both reported.

5. Experimental results

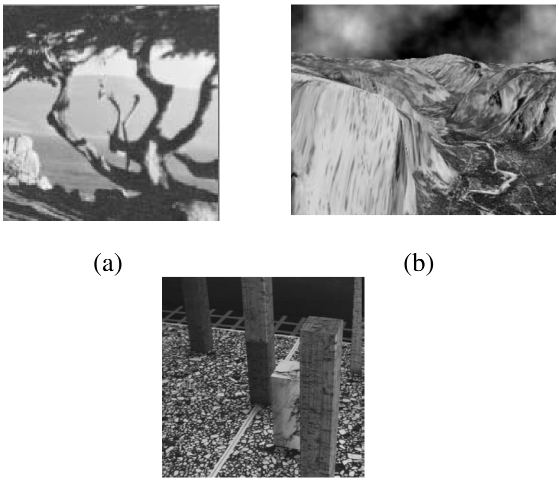
The proposed algorithm has been evaluated on both synthetic and real images. Three well-known image sequences (the Diverging Tree sequence[2]; the Yosemite sequence[2]; and the Otte[3] image sequence) are used (see figure 3).Table 1 shows the comparison results the Diverging Tree sequence (figure 3 (a)) – showing the proposed method gives the most accurate results for affine motion model. Even for the constant motion model, vbQMDPE still yields better results than most other comparative methods.

Figure 3 (b) shows one snapshot of the Yosemite sequence. Because the true motion of the clouds does not really reflect the image brightness changes, we exclude the clouds in our experiments. From table 2, we can see that the proposed algorithm and Farneback's

[2] The sequences are obtained from ftp://csd.uwo.ca/pub/vision

[3] The sequence is obtained from http://i21www.ira.uka.de/image_sequences/

algorithms give the best overall results. The standard variance error of our results is less than that of Farneback's results (2000, 2001) for both constant and affine motion models. Although the averaged angle error of Farneback's results (2000) is better than our results for constant motion model, our results for affine motion model with larger local window outperform Farneback's results (2000). However, the average angle error of Farneback's later version (2001), which used an affine motion model and a combined a region growing segmentation algorithm, is better than ours. To our knowledge, it is the best result obtained so far in the field of optical flow computation for Yosemite sequence.



(a) (b) (c)

Figure 3. The snapshot of the three image sequences: (a) the Diverging Tree; (b) the Yosemite; and (c) the Otte sequence.

We also note that our results with affine motion model are better than those with constant motion model in both the Diverging Tree and the Yosemite. This is because the motion in two sequences is mostly diverging. For each pixel within a small local window, the optical flow changes. Thus, the affine motion model reflects the true situation better than the constant one.

The Otte sequence (figure 3 (c)) is a real image sequence [27] and it is difficult because it includes many sharp discontinuities in both motion and depth. When we recomputed the optical flow for Otte image sequence (frame 35) by Bab-Hadiashar and Suter's code, we found that the results in [1] were wrongly reported (our results show an improved performance!). From table 3, we can see our results outperform all other published.

6. Conclusions

We have developed a novel robust estimator—variable bandwidth QMDPE, and we applied it to optical flow computation. By employing nonparametric density estimation and density gradient estimation techniques in parametric model estimation, the proposed method is very robust to outliers and is a substantial improvement over traditional methods. We expect we can do even better with a multi-resolution version of our approach.

Our code *without optimization* takes about 6 min on Yosemite image sequence on a 1.2GHz AMD personal computer, using 17x17 patches around each pixel and m is set to 30. The speed can be improved for less m and smaller patches but with worse accuracy. The mean number of mean shift iterations is about 3 for each p-subset.

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Table 1. Comparative results on diverging tree: the first part of the table is the results reported by Barron et. al (1994) and Ong et. al (1999); the second part is the results obtained by the proposed algorithm (number 2 and 6 represent constant and affine motion models)

Technique	Avg. error (degree)	Std. dev. (degree)	Density (%)
Horn and Schunck (original unthresholded)	12.02	11.72	100
Horn and Schunck (modified unthresholded)	2.55	3.67	100
Uras et.al. (unthresholded)	4.64	3.48	100
Nagel	2.94	3.23	100
Anandan	7.64	4.96	100
Singh (Step 1 unthresholded)	17.66	14.25	100
Singh (Step 2 unthresholded)	8.60	4.78	100
Least-Squares (block-based) method (in Ong and Spann, 1999)	1.98	2.81	100
vbQMDPE2 (σ_0=1.5, 11x11, m=30)	2.51	1.62	100
vbQMDPE6 (σ_0=1.5, 11x11, m=30)	1.46	1.03	100

Table 2. Comparative results on Yosemite (cloud region excluded): the first part is the results reported in the recently referenced literature; the second part is our results.

Technique	Avg. error (degree)	Std. dev. (degree)	Density (%)
Black (1994)	3.52	3.25	100
Szeliski and Coughlan (1994)	2.45	3.05	100
Black and Anandan (1996)	4.46	4.21	100
Black and Jepson (1996)	2.29	2.25	100
Ju et. al. (1996)	2.16	2.00	100
Memin and Perez (1998)	2.34	1.45	100
Memin and Perez (2002)	1.58	1.21	100
Lai and Vemuri(1998)	1.99	1.41	100
Bab-Hadiashar and Suter (WTLS2, 1998)	2.56	2.34	100
Bab-Hadiashar and Suter (WTLS6, 1998)	1.97	1.96	100
Farneback2 (2000)	1.94	2.31	100
Farneback6 (2000)	1.40	2.57	100
Farneback6 (2001)	1.14	2.14	100
vbQMDPE2 (σ_0=2.0, 17x17, m=30)	2.12	2.08	100
vbQMDPE6 (σ_0=2.0, 17x17, m=30)	1.54	1.99	100
vbQMDPE2 (σ_0=2.0, 25x25, m=30)	2.27	2.07	100
vbQMDPE6 (σ_0=2.0, 25x25, m=30)	1.34	1.69	100

Table 3. Comparative results on Otte image sequences: the first part was reported by Bab-Hadiashar and Suter (1998); the second part is the corrected results; the third part is obtained by running the proposed algorithm.

Technique	Avg. error (degree)	Std. dev. (degree)	Density (%)
Giachetti and Torre (1996)	5.33	-----	100
Bab-Hadiashar and Suter (WLS2, 1998)	3.39	6.55	100
Bab-Hadiashar and Suter (WLS6, 1998)	3.51	6.48	100
Bab-Hadiashar and Suter (WTLS2, 1998)	3.74	8.09	100
Bab-Hadiashar and Suter (WTLS6, 1998)	3.67	7.37	100
Bab-Hadiashar and Suter (WLS2, corrected)	3.02	5.98	100
Bab-Hadiashar and Suter (WLS6, corrected)	3.14	5.84	100
Bab-Hadiashar and Suter (WTLS2, corrected)	3.20	7.02	100
Bab-Hadiashar and Suter (WTLS6, corrected)	3.20	6.59	100
vbQMDPE2 (σ_0=2.0, 17x17, m=30)	2.64	4.98	100
vbQMDPE6 (σ_0=2.0, 17x17, m=30)	2.82	5.03	100
vbQMDPE2 (σ_0=2.0, 25x25, m=30)	2.21	4.16	100
vbQMDPE6 (σ_0=2.0, 25x25, m=30)	2.29	4.06	100