Building Roadmaps of Local Minima of Visual Models

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Contributions

- Optimization-based algorithms for **systematically finding nearby local minima** of high-dimensional cost functions, and building 'roadmaps' of the low-peak-cost paths linking them.
  - No prior knowledge of minimum distribution or cost topography needed.
  - Can also handle constraints (inequality, non-linear).

Approach

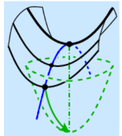
- Adapt computational chemistry methods for finding ***Transition States***
  - codimension 1 saddle points (0 gradient, *1* negative curvature direction)
- Two deterministic approaches based on local optimization:
  - ***Eigenvector tracking:*** modified form of damped Newton minimization.
  - ***Hypersurface sweeping***: propagates a hypersurface through the space, tracking minima on it.
- From each transition we slide downhill to next minimum
  - minimum peak cost pathway linking the minima

Conclusions

➔ Efficient deterministic algorithms for exploring high-dimensional non-convex likelihood surfaces.

➔ Useful for many other vision problems (model-based vision, SFM).

Eigenvector Tracking

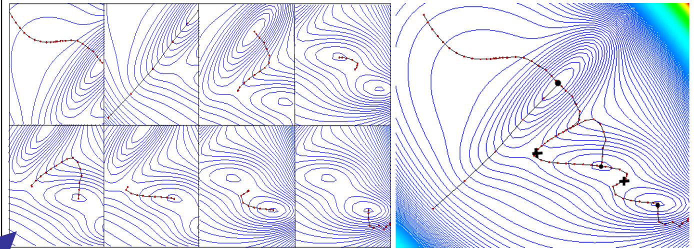


- Choose an ascent eigenvector *'k'* and track it as you progress. Move uphill along *'k'* and downhill along all other eigenvectors
- Modify damping signs to minimize an equivalent local virtual cost model

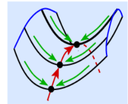
$$\delta x(\lambda)=\left(\frac{g_1}{\lambda_1+\sigma_1\lambda},\dots,\frac{g_n}{\lambda_n+\sigma_n\lambda}\right)^T,\quad \lambda\geq-\min(0,\sigma_1\lambda_1,\dots,\sigma_n\lambda_n),+\infty),\quad \sigma_k=-1,\sigma_{i\neq k}=+1$$

➔ Tracking 'a given eigenvalue' is ill-defined : when eigenvalues cross, their eigenvectors slew around rapidly.

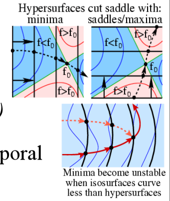
➔ Seach may diverge to infinity (wall-climbing).



Hypersurface Sweeping



- A codimension 1 saddle is
  - a local maximum in one direction
  - a local minimum in the other *n-1* directions
- Sweep space with a moving hypersurface
  - hyperplane, expanding hyper-ellipsoid…
  - parametrized implicitly *c(x)=t* or explicitly *x=x(u,t)*

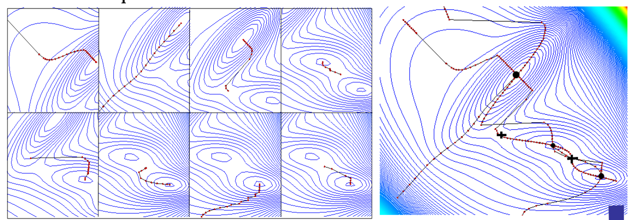


- Track local-minima within hypersurface and find temporal maxima along tracks

➔ Only finds saddles that surface cuts in positive curvature directions

➔ Some ('sawtooth') maxima are disappearances not saddle points

➔ Plane / ellipse orientation fixes initial track directions.



Local Minimization versus Saddle Point Search

- ***Minimization***
  - many local descent based algorithms
  - measure progress by decrease in function value: local 'sufficient decrease' ensures 'global convergence' to some minimum
- ***Saddle Point Search***
  - ascend towards saddle using modified descent algorithms
  - no universal progress criterion
  - needs initial ascent direction or direction weighting

Newton Minimization

- Pure Newton minimization (f = cost function, g = gradient, H = Hessian)

$$0=g(x+\delta x)\approx g(x)+H\,\delta x \quad\Rightarrow\quad \delta x=-H^{-1}g$$

  - efficient near minimum
  - further away, may diverge or converge to any type of stationary point.
- Damped Newton minimization
  - globalize convergence by adding 'damping' matrix D

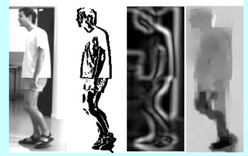
$$\delta x=(H+\lambda D)^{-1}g$$

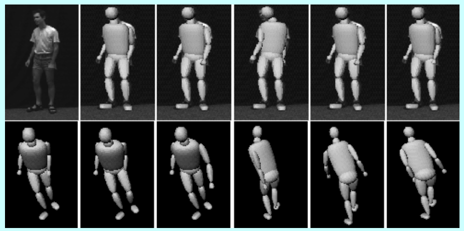
  - in an eigenbasis where $D=I$:

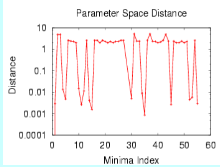
$$\delta x(\lambda)=\left(\frac{g_1}{\lambda_1+\lambda},\dots,\frac{g_n}{\lambda_n+\lambda}\right)^T,\quad \lambda\geq-\min(0,\lambda_1,\dots,\lambda_n)$$

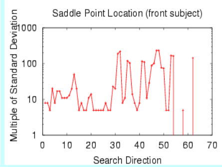
Application: Monocular Human Pose, Reconstruction and Tracking Ambiguities

- 32 dof articular human model with superquadric parts, joint angle limits and part non-interpenetration constraints
- Initialize using model-image joint correspondences, track using edge+silhouette and edge+flow cost.
- Method finds many (~300) minima caused by ambiguous (forwards/backwards) depths or incorrect assignments.
- Many more minima are eliminated by physical (joint, non-interpenetration) constraints









Pose ambiguities for given model-image joint correspondences (frontal and half-profile views), and some statistics on minima found.

Tracking Ambiguities: flow+edge (top) and silhouette+edge (middle & bottom) cost function