COLOR MODE FILTERING

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ABSTRACT

In this paper mode filtering of color images is explored. An existing framework based on local histograms is extended to multi-channel images. Within this framework three color mode operations are proposed; 1. global mode operation for edge sharpening, noise reduction and small object removal, 2. constrained mode operation for white noise filtering while preserving detail, and 3. uncertain data mode filtering to incorporate prior knowledge about the certainty of the measurements into the mode computation. Results obtained for a variety of images indicate the feasibility of color mode filtering.

1. INTRODUCTION

Extending image processing operations from one-channel to multichannel operations often fails due to the absence of a natural basis for vector ordering. We will call this problem the ordering problem. Applying an operation to the channels separately and subsequently combining the results, ignores the correlation between the channels. As a result artefacts may be introduced, e.g. new chromaticities in the case of color images [1]. In this paper we focus on color images.

An approach is to use an adapted version of median filtering to avoid the problem of ordering (see e.g. [1] , [2]). This is established by defining the median of a set as the element of the set which has the least summed distance to all other elements.

In this paper, we propose the mode operation to avoid the ordering problem. We focus on global mode filtering (highest mode of a distribution). The global mode of a distribution is invariant for injective re-mapping of the values. As a consequence the mode filter is suitable for images without global topology. In our application we will focus on these globally unordered images, such as document images, cartoon images, etc. Examples of ordered images are light microscopy and geodesic images, where there is also a clear ordering of the measurements, e.g. less- or more light absorbing cell-tissue.

To present color mode filtering in a principle way, the imprecision space is used. Griffin [3] proposes this framework of working with local histograms at different scales. Next to a spatial scale, an imprecision scale is introduced at which the local histogram is observed, also referred to as the ”tonal scale”. The main focus of Griffin is on the evolution of the median and the stable mode to find perceptual edges.

Further, Koenderink et al. [4] extend the imprecision space with a third scale, the inner scale. This is the scale at which the image is observed. The combination of the three scales are called locally orderless images. Locally orderless because a global but not a local topology is defined. Van Ginneken et al. [5] have considered a number of applications on grey scale information based on such locally orderless images.

The paper is organised as follows. In section 2 the local histogram framework is explained and a global mode operations, a constrained mode operation and a mode operation for uncertain data are proposed. In section 3 details about the implementation are given. Section 4 gives conclusions.

2. LOCAL HISTOGRAM SPACE

In this section the theory of Griffin [3] is explained and extended to multi-channel images. The local histogram space (LHS), is a combination of the spatial space and the sensor space. In fact a local histogram is constructed for every position in the image.

To be precise, given an image

$$f(.) : R^n \rightarrow R^m \tag{1}$$

where n is the spatial dimension and m the dimension of the sensor space. The LHS at spatial-scale and sensor-scale zero is defined as

$$H : R^n \times R^m \rightarrow R \tag{2}$$

$$H\left(x, i, \sigma_x = 0, \sigma_i = 0\right) = 1 \Leftrightarrow f(x) = i \tag{3}$$

This result is filtered with an $n + m$ dimensional Gaussian filter to obtain the LHS at spatial scale σ_x and a sensor scale σ_i

$$H(x, i, \sigma_x, \sigma_i) = H(x, i, 0, 0) \otimes (G(x, \sigma_x) \cdot G(i, \sigma_i)) \tag{4}$$

with

$$G(x, \sigma) = \frac{1}{(2\pi)^{\frac{n}{2}} \sigma^n} e^{-\frac{1}{2}\left(\frac{x^T \cdot x}{\sigma^2}\right)} \tag{5}$$

For reasons of clarity σ is taken as a scalar. Extension to a vector is straightforward.

The LHS is dependent on the two scale parameters of the two scale spaces involved: the spatial scale σ_x and the sensor scale σ_i. The spatial scale σ_x regulates the size of the local neighborhood of the local histogram. Changing from zero, only the current pixel is represented in the local histogram, to ∞ in which case the LHS at every position is equal to the global histogram. The sensor space scale σ_i is the scale at which the local histogram is smoothed. For unordered images, σ_i can be interpreted with respect to the topology of the image. As mentioned these images have no global topology but there is some local topology due to imperfections of the surface and limitation of the acquisition system. σ_i determines which sensor space values are still considered "the same".

2.1. Global Mode Projection

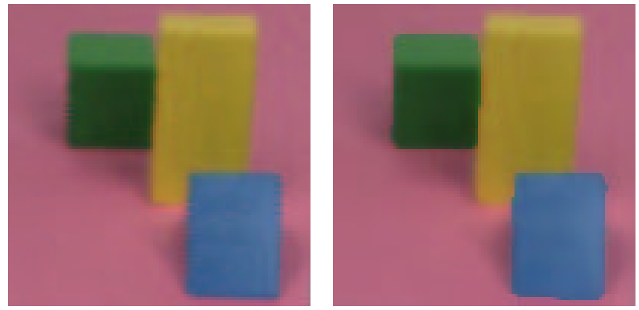
The maxima in a local distribution are called modes. In this paper we will only focus on the highest mode, called the global mode. For a histogram this is easily computed by finding the most filled bin. For LHS this is done by maximum projection of the LHS on the spatial plane

$$m(x, \sigma_x, \sigma_i) = \arg\max_i (H(x, i, \sigma_x, \sigma_i)) \tag{6}$$

Interpreting the global mode projection from a filtering point of view leads to the following explanation of LHS. Applying the classical definition, the mode is the most common value of a set. Consequently, the mode for small sets with noise (e.g. a local neighborhood in an image) is a very noise dependent operation. To reduce the noise dependence we allow for some uncertainty in the values of the set. A value is partially related to neighboring values. This is done by smoothing the sensor space with a Gaussian kernel and thereby stabilizing the mode operation.

Figure 1 shows an example of edge sharpening by mode operation [1]. Due to the limited resolution of the optical system pixel values on edges are mixtures of the values on both sides of the edge. Removal of these pixels is desired as a preprocessing step for several image operations. As can be seen the pixels on color transitions are assigned to one of the two colors. Mixtures between the colors which where present in the input image have been removed.

[1] see section on implementation for explanation of β



(a) (b)

Fig. 1. a) input image (100x100) b) global mode of input image with $\sigma_x = 1.5, \sigma_i = 16$ and $\beta = 16$

In figure 2 an application of LHS mode filtering is shown where, objects which are relatively small compared to the the spatial scale of the LHS, are removed. In figure 2c the local histogram at two spatial scales is shown. The position of the local histogram is indicated with an arrow in the image. At the small scale the character-color still determines the global mode. However with increasing spatial scale the influence of the background increases. At a certain scale the mode will jump to the background color, see figure 2b.

2.2. Constrained Mode Filtering

Pixels which are spatially further away have less influence on the local histogram. In this section we expand this concept to the sensor space. Consequently, pixel values which are further away of the current pixel value in sensor space have less influence on the local histogram. This could also be interpreted as a cost-function for the amount of change caused by the filter. An advantage of such a filter is that it reduces the noise while it preserves detail. The implementation is done by introducing a third scale, σ_c

$$H_c(x, i, \sigma_x, \sigma_i, \sigma_c) = H(x, i, \sigma_x, \sigma_i) \cdot G(i, \sigma_c) \tag{7}$$

The choice for σ_c is mainly dependent on the noise and the minimal contrast between different colors. Other noise distortions like e.g. speckle noise, are not removed by constrained mode filtering.

In figure 2d the constrained histogram of figure 2c is given. The constraint mode still returns the color of the characters, even for the larger scale. However at the larger spatial scale the pixels where averaged with more other character pixels and better noise reduction is obtained while preserving the characters (the results for constrained mode filtering are not given for the exclamation mark image).

In figure 3 uncorrelated Gaussian noise is added to a document image. By constrained mode filtering the noise is

(a) (b) (c) (d)

Fig. 2. a) input image (230x160) b) global mode of input image with $\sigma_x = 3, \sigma_i = 16$ and $\beta = 16$ c) local histogram at two scales (projected with $i = R + G + B$ for visualisation purposes) of character in input image, $\sigma_i = 8$, location character is indicated with arrow. d) constraint mode with $\sigma_c = 4$ of histogram of c).

(a) (b) (c) (d)

Fig. 3. a) input image (150x130) b) corrupted with uncorrelated Gaussian noise $\sigma = 20$ c) constrained mode with $\sigma_x = 4$, $\sigma_i = 48$, $\sigma_c = 1.75$ and $\beta = 16$ d) vector median (3x3 neighborhood) as proposed in [1]

reduced while details are preserved without smoothing the edges. For comparison the vector median as proposed in [1] is given in 3d. For the median an apparent smoothing effect is visible.

2.3. Incomplete and Uncertain Data Mode Filtering

Knutsson [6] proposes a method for filtering grey scale images with incomplete and uncertain data. This method can be extended to multi-channel images by using the LHS framework.

σ_i is introduced as one of the scale parameters of the LHS. The functionality of σ_i can be broadened by incorporating knowledge of the uncertainty of the data. This results in a smaller σ_i for certain data than for uncertain data. Then we obtain for eq. 2

$$H(x, i, \sigma_x, \sigma_i(x, i)) = H(x, i, 0, 0) \otimes (G(x, \sigma_x) \cdot G(i, \sigma_i(x, i))) \quad (8)$$

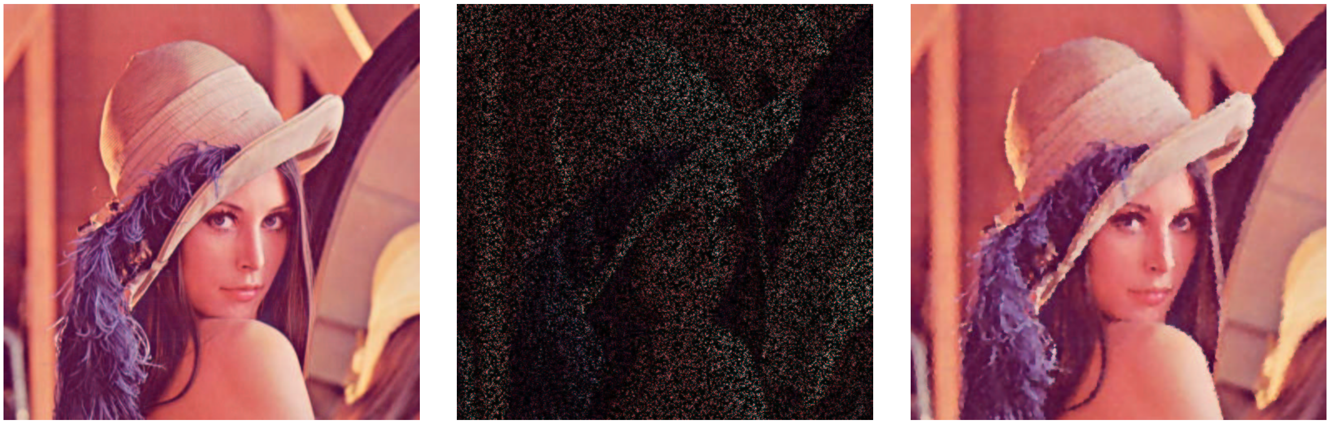
where $\sigma_i(x, i)$ denotes the uncertainty in sensor space.

In figure 4 an example of missing data mode filtering is given. 15% of the image pixels were randomly selected. On the position of missing data we have $\sigma_i(x) = \infty$. The mode projection of this LHS is given in figure 4c. The reconstruction image still contains sharp edges and details, like the texture of the feather.

We mention two examples where knowledge of the uncertainty can be used within the local histogram framework. One example is a filtering operation that operates in a transformed colorspace. In such a case the known noise distribution in the R,G,B-space must be propagated to the new colorspace. This information can then be used for choosing σ_i. An example of dependence of σ_i on i is data which main noise pollution is caused by Poisson noise.

3. IMPLEMENTATION

To obtain efficient implementation of the 5D LHS for color image mode filtering, we propose the following technique to reduce memory usage. For this purpose the resolution of the sensor space is reduced.



(a) (b) (c)

Fig. 4. a) original Lena image (512x512) b) 15 % of the pixels of te input image randomly chosen. c) global mode interpolation of b) with $\sigma_x = 1.5, \sigma_i = 2.5, \beta = 16$

The LHS at a certain position x_c is computed as follows

$$H\left(x_c, i_\beta, \sigma_x, \sigma_i\right) = \int e^{-\left(\frac{(x_c - x)(x_c - x)^T}{2\sigma_x^2} + \frac{\left(i_\beta - f(x)\right)\left(i_\beta - f(x)\right)^T}{2\sigma_i^2}\right)} dx \tag{9}$$

in which the elements of the vector i_β are taken from the following set

$$\left\{0, \frac{1}{\beta} i_{\max}, \frac{2}{\beta} i_{\max}, ..., \frac{\beta - 1}{\beta} i_{\max}\right\} \tag{10}$$

After finding the maximum value in the reduced local histogram the actual mode is estimated by a parabolic fit in the neighborhood of the maximum.

For color images a 3D parabola is fitted to the data. The parabolic least-square fit in the local histogram is based on the following equation

$$H\left(i = \{R, G, B\}\right) = A_o + A_1 \cdot \left((R - R_m)^2 + (G - G_m)^2 + (B - B_m)^2\right) \tag{11}$$

rewriting in the form $I\,B = H$ with

$$I = \begin{pmatrix} 1 & R_1 & G_1 & B_1 & R_1^2 + G_1^2 + B_1^2 \\ . & . & . & . & . \\ 1 & R_7 & R_7 & R_7 & R_7^2 + G_7^2 + B_7^2 \end{pmatrix} \tag{12}$$

$$B = \begin{pmatrix} A_0 + A_1(R_0^2 + G_0^2 + B_0^2) \\ -2 \cdot A_1 \cdot R_0 \\ -2 \cdot A_1 \cdot G_0 \\ -2 \cdot A_1 \cdot B_0 \\ A_1 \end{pmatrix} \tag{13}$$

A six connected neighborhood generates an overdetermined system of 7 equations for 5 parameters. The maximum of the parabola (R_m, G_m, B_m) is returned as the mode of the local histogram.

4. CONCLUSION

In this paper a new method for color filtering is proposed based on the mode operation. As a basis the local histogram space is used. Three color mode filters are proposed and have been applied successfully for edge sharpening, noise reduction while preserving detail, small object removal and missing data interpolation. Further research is needed to relate the proposed filters with existing color filters in literature.

5. REFERENCES

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