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Brief review of invariant texture analysis methods ☆

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Abstract

This paper considers invariant texture analysis. Texture analysis approaches whose performances are not affected by translation, rotation, affine, and perspective transform are addressed. Existing invariant texture analysis algorithms are carefully studied and classified into three categories: statistical methods, model based methods, and structural methods. The importance of invariant texture analysis is presented first. Each approach is reviewed according to its classification, and its merits and drawbacks are outlined. The focus of possible future work is also suggested.

Keywords: Invariant texture analysis; Statistical methods; Model based methods; Structural methods

1. Introduction

Texture analysis is a basic issue in image processing and computer vision. It is a key problem in many application areas, such as object recognition, remote sensing, content-based image retrieval and so on. Texture analysis has been an active research topic for more than three decades. Numerous methods have been proposed in the open literature. Surveys of existing texture analysis approaches may be found in Refs. [1–8]. Comparative studies of a subset of texture analysis methods can also be found in Refs. [30–33]. The importance of texture perception is presented in detail [1] from the viewpoint of human vision and practical machine vision applications.

However, the majority of existing texture analysis methods make the explicit or implicit assumption that texture images are acquired from the same viewpoint (e.g. the same scale and orientation). This gives a limitation of these methods. In many practical applications, it is very difficult or impossible to ensure that images captured have the same translations, rotations or scaling between each other. Texture analysis should be ideally invariant to viewpoints. Furthermore, based on the cognitive theory and our own perceptive experience, given a texture image, no matter how it is changed under translation, rotation and scaling as well as affine transform or even perspective distortion, it is always perceived as the same texture image by a human observer. Invariant texture analysis is thus highly desirable from both the practical and theoretical viewpoint.

More and more attention has been paid on invariant texture analysis. A great deal of work has been done on this important topic. Survey of the existing work on invariant texture analysis is highly desirable for new comers in this area. However, few existing survey papers discuss this subject despite its growing importance (e.g. in content-based viewpoint invariant image retrieval). This paper attempts to fill this gap by presenting an overview of the existing work on the subject in an

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attempt to facilitate more research on this important issue.

Existing texture analysis methods are broadly divided into two categories: statistical methods and structural methods in most of the surveys [1–8]. But in the recent literature, many model based methods have been employed in texture analysis [10], including autoregressive model, Gausian Markov random fields, Gibbs random fields, Wold model, wavelet model, multichannel Gabor model and steerable pyramid, etc. These models provide more powerful tools for invariant texture analysis. Since they have many similar properties, we classified these methods into a relatively independent category to present a clearer classification.

The methods covered in this paper are therefore broadly divided into three categories: statistical methods, model based methods, and structural methods. The importance of the analysis of the coordinate transform is first emphasized to state the task of invariant texture analysis. In the following sections, overviews of the existing work on translation, rotation and scale invariant texture analysis are presented, with the principles, merits, and weaknesses of each approach outlined. It should be pointed out that the experiments of various methods reported in this paper are conducted by the authors of the work being reviewed.

2. Coordinate system transform

Coordinate transforms play a very important role in invariant texture analysis. So some problems of these coordinates are first discussed in this section.

Translations, rotations and scaling of texture images are caused in nature by a linear transform of the image points coordinates. Many invariant texture analysis methods exploit some coordinates transform to get invariant texture features. Analysis of the transform properties is necessary. The geometric transform of texture images can be described as the following mathematical model (here we assume affine transform):

$$\begin{bmatrix} x_a \\ y_a \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}, \quad (1)$$

$$f_a(x_a, y_a) = f(x, y), \quad (2)$$

where the function $f(x, y)$ denotes the original image and $f_a(x_a, y_a)$ the transformed image. Let matrix $C_{2\times 2}$ denote

$$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \quad \text{and} \quad D_{2\times 1} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix},$$

where $D_{2\times 1}$ is the translation factor, and $C_{2\times 2}$ is the rotation, scaling and skew factor. If $C_{2\times 2}$ is an orthogonal matrix, this implies that in Eq. (1) only translation and rotation occur. It is obvious that the essential task of invariant texture analysis is how to extract the texture features which are not affected by geometric transforms such as the affine transform.

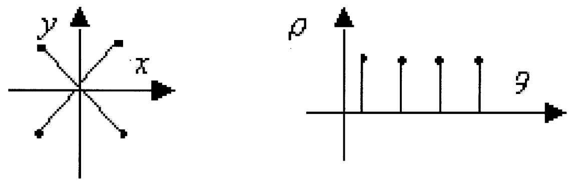

Fig. 1. Rotation of the four points in the Cartesian grid (a) maps to translation in the polar coordinates system (b).

One way to handle rotation and scaling is to use a suitable transform such that rotation and scaling will appear as a simple displacement in the new coordinates system. Employing the polar coordinates system or log-polar grid is a very popular approach to solve this problem [73,74,79,96]. It is well known that rotation in the Cartesian grid (x, y) corresponds to a circular translation along the axis in the polar grid (ρ, ϕ) as Fig. 1 shows. Also scaling in the (x, y) grid corresponds to a shift along the $\log \rho$ axis in the log–polar grid $(\log \rho, \phi)$. The drawback of applying these methods for invariant analysis is that the translation invariance is sacrificed. So some shift-invariant methods must be applied in advance (for example the Fourier transform).

Some examples of texture geometric transform are illustrated in Fig. 2. They clearly indicate that although the orientation and scale of textures have been changed, the images give us the perceptions of the same textures.

3. Statistical methods

In statistical methods, texture is described by a collection of statistics of selected features. Many of these methods are based on Julesz's finding [78,81] that the human visual system uses statistic features for texture discrimination including the first-order statistics, the second- order statistics and the higher-order statistics. But these properties may not make the same contribution in the description of the texture when humans use them in visual recognition. Accordingly, the statistics are broadly classified into first-order statistics, second-order statistics, and higher-order statistics. Many authors have developed those statistical properties for invariant texture analysis. In the early 1980s about the time Davis did the polarogram work, some efforts on texture analysis were made to achieve texture invariance [5,8]. In the following subsections, we will address the existing invariant texture methods based on statistics.

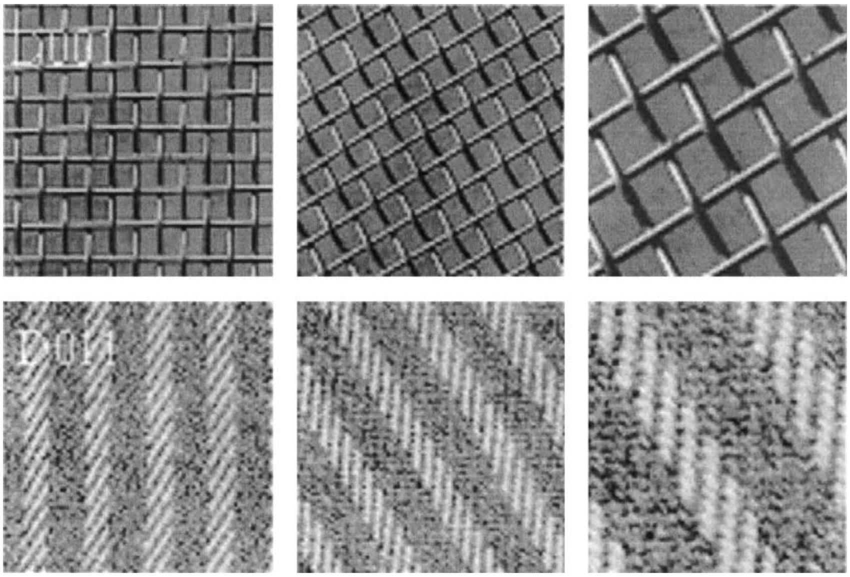

Fig. 2. Examples of texture geometric transform (here we assume that only rotation and scaling occur).

3.1. Polar plots and polarograms

Davis [11] describes a new tool (called polarogram) for image texture analysis and used it to get invariant texture features. A polarogram is a polar plot of a texture statistics as a function of orientation. Let $D(a)$ be a displacement vector of fixed magnitude d and variable orientation a. A polarogram is then defined as follows:

$$P_f(a) = f(C_{D(a)}), \tag{3}$$

where $C_{D(a)}$ is the co-occurrence matrix for displacement $D(a)$, f is some statistics such as contrast, defined for a co-occurrence matrix. Similarly, polarograms can easily be defined on texture features other than those derived from co-occurrence matrices.

Texture features are derived from the polarogram by computing size and shape features of the polarogram. The shape features not only depend on the shape of the boundary of the polarogram, but also depend on the position of the origin of the polarogram. Note that the features computed from the polarogram are invariant to the orientation of the polarogram. Since the rotation of the original texture results in the rotation of the polarogram, the features are invariant to the rotation of the original texture.

Two experiments on a database of five texture classes at 16 orientations are performed to test the use of the polarogram statistic for texture classification. In the experiments, the polarogram is obtained from the contrast descriptors of the gray level co-occurrence matrix. In the first experiment, the textures are classified by employing a multivariate linear discriminant function to obtain a correct classification rate ranging from 75% to 90%.

In Davis's method, the co-occurrence matrix of a texture image must be computed prior to the polarograms. However, it is well known that a texture image can produce a set of co-occurrence matrices due to the different values of a and d. This also results in a set of polarograms corresponding to a texture. Only one polarogram is not enough to describe a texture image. How many polarograms are required to describe a texture image remains an open problem.

3.2. Texture edge statistics on polar plots

The polar grid is also used by Mayorga and Ludeman [12] for rotation invariant texture analysis. The features are extracted on the texture edge statistics obtained through directional derivatives among circularly layered data. Two sets of invariant features are used for texture classification. The first set is obtained by computing the circularly averaged differences in the gray level between a pixel. The second computes the correlation function along circular levels.

Neural nets are used here to classify the image on a pixel by pixel basis. An 8-nearest neighbor clustering procedure is applied to the output from the net with the goal of grouping the classified 3×3 sections into larger regions.

For demonstrating the classification performance of the neural nets, their algorithm is tested on a set of four textures selected from the Brodatz album. Each texture is rotated at four orientations: $0^\circ, 45^\circ, 90^\circ$, and 135°.

The window size 11×11 is applied for the selection of the training patterns on the assumption that it contains enough information of the texture. A correct classification rate of 75% was obtained on the selected textures. It is obvious that this method may produce better results applied to those textures with strong texture edges than those with random properties. Also the texture features definitely depend on texture edge detection method.

3.3. Harmonic expansion

Harmonic expansion and Mellin transform have been widely shown to be an efficient approach towards invariant pattern recognition [13–15,86–89,91,92,94]. In this approach, an image is decomposed into a combination of harmonic components in its polar form. The projection of the original image to the harmonic operators gives the features of the pattern. The magnitudes of these coefficients are used for rotation invariant pattern analysis.

Alapati and Sanderson [15] describe an application of a set of 2-D multi-resolution rotation invariant operators (MRI) for texture classification based on harmonic expansion. A complex kernel in polar coordinates defines a class of rotation invariant operators as follows:

$$H_n(r,\varphi) = h(r)e^{im\varphi}, \quad m = 0, 1, 2, \ldots, \tag{4}$$

where $h(r)$ is a one-dimensional function of the radius r. The function set $\{H(r,\varphi)\}$ composes an orthogonal set of complex masks due to the orthogonality of $e^{jm\varphi}$. Convolution of an operator with a texture produces a complex output texture containing magnitude and phase information. Application of a set of such operators at different resolutions to a texture yields an expansion of texture outputs. The magnitude derived from the expansion is used as the texture features invariant to rotations.

The algorithm is tested on four structurally similar textures selected from the Brodatz database [72]. The coefficients of the series expansion are used for texture classification. Texture classification is performed with a better than 90% accuracy.

This method proposed by Alapati and Sanderson is only invariant to rotation. In fact, their method can be considered as the Fourier transform in the polar grid and can be developed to deal with scaling invariance. If we employ log operator on the radius r and then apply the Fourier transform along the log version of r, scale invariant texture features may also be obtained.

3.4. Optical digital method

Duvernoy [16] uses the technique of Fourier descriptors to extract the invariant texture signature on the spectrum domain of the original texture. In his method, the image is characterized by the optical Fourier spectrum of the texture. A contour line is defined by selecting the points whose energy equals to a given percentage of the central maximum of this spectrum. The energy of the coefficients of the descriptors is invariant to the rotation and change of the scale of the curve. A faster expansion based on Legendre polynomials is used to reduce the dimensionality of the descriptors.

Experiments are performed on the restricted three classes of the textures associated with fields, woods, and urban regions in remote sensing images. The results show that this method is efficient and is able to discriminate the three classes of textures accurately. But the drawback of this method is the difficulty in determining the contour line on the spectrum domain. Different thresholding algorithm results in different shapes of the contour line, which in turn affects the extraction of the texture signature.

3.5. High-order statistics

High-order statistics have been already involved in the texture analysis of the early years by Real et al. (1972), and Galoway (1975) [3]. These include interval cover, a series of statistics on the run length methods, and triple correlation methods.

Tsatsanis and Giannakis [108] employed the high-order statistics (cumulant and multiple correlation) to solve the invariant texture classification and modeling problems. An energy detector is developed in the cumulant domain. It is demonstrated as being immune to Gaussian noise and insensitive to object translation.

Experiments are performed for texture classification both in the random and in the deterministic cases. Four test cases are considered for application in texture analysis.

Test cases #1 and #2 are designed for evaluating the immunity of the cumulant to additive Gaussian noise under various signal noise ratio (SNR) values. The result of test case #1 shows that the cumulant almost keeps the same value even for $SNR = -4$ dB compared with the bispectrum-based detector proposed by Hinich [42]. Test case #2 demonstrates that this algorithm also works very well on invariant character recognition and the results are still satisfactory for $SNR = 8$ dB. In text case #3, a comparison of this method with an auto-correlation classifier is reported. A database containing 32 images generated from 8 Brodatz textures is used. A correct classification rate up to 100% is obtained due to the use of the optimal normalized cumulant. The advantage of the method is shown in case #4 when applied for the phase analysis of texture. The drawback of this method is its high computational cost.

Tsatsanis and Giannakis [108] have demonstrated in their experiment that using the high-order statistics can give a promising result, however Julesz [78] has suggested that there may be little useful information in high-order statistics for texture discrimination. So this contradiction produces an interesting problem. Whether high-order statistics can provide a very powerful tool for texture discrimination needs further study.

3.6. Moment invariants

Moment of the image is another powerful statistics tool for pattern recognition [9,54]. It is defined as the projection of a function $f(x, y)$ onto a monomial $x^p y^q$:

$$M_{pq} = \int\int x^p y^q f(x, y)\,\mathrm{d}x\,\mathrm{d}y. \tag{5}$$

Many normalization methods have been proposed to obtain different forms of the moment invariants [53–55]. It has been demonstrated by Teh and Chin [101] that Zernike moments are the best compared to the other four different moments (Legndre, Complex, Rational, and regular).

Zernike moments are the projection of the function $f(x, y)$ onto Zernike polynomials:

$$z_{mm} = [(n+1)/\pi] \int\int_U f(x, y) V^*_{nm}(x, y)\,\mathrm{d}x\,\mathrm{d}y, \tag{6}$$

Zernike polynomials $V^*_{nm}(x, y)$ are one of an infinite set of polynomials that are orthogonal over the unity circle U: $x^2 + y^2 \leqslant 1$, i.e.

$$\int\int_U V_{pq}(x, y) V^*_{nm}(x, y)\,\mathrm{d}x\,\mathrm{d}y = \frac{\pi}{n+1}\delta_{pn}\delta_{pm}$$

with

$$\delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

The orthogonality of Zernike moments provides an important property: the magnitudes of Zernike moments are invariant to rotation (for details see Ref. [52]). It is demonstrated by many recent publications that Zernike moments perform well in practice to obtain geometric invariance. Wang and Healey [52] develop this method for texture analysis. In their method, texture is characterized using Zernike moments of multispectral correlation functions. A relationship is established between the correlation functions under different geometric and illumination conditions to get the invariance. Scale invariance is obtained by the normalized correlation functions. The correlation function is invariant to translation in nature.

An experiment on a database of 7 textures is described to test the performance of this method for invariant texture classification. A total of 224 test images are constructed by eight translated, rotated, and scaled versions of each texture imaged under each of four illuminations (white, yellow, red, and green). The correlation function is estimated and transformed to the polar coordinates by an interpolation. Zernike moments are extracted from each correlation to form a set of translation and rotation invariants. The highest classification rate of 100% is reported on these 224 test textures.

3.7. Feature distribution method

Pietikainen et al. [110] present some features based on center-symmetric auto-correlation, local binary pattern, and gray-level difference to describe texture images. Most of these features are locally invariant to rotation including linear symmetric auto-correlation measures (SAC), rank order versions (SRAC), related covariance measures (SCOV), rotation invariant local binary pattern (LBPROT) features and gray-level difference (DIFF4) (see Ref. [110] for more details). A feature distribution method is proposed based on the G statistics to test those features for rotation invariant texture analysis. An experiment is performed on 15 Brodatz textures to test this method in two cases. In the first case, each texture image undergoes rotation angles at 30°, 60°, 90°, 120° and 200°. Since a single feature does not contain enough information to describe a texture, the experiment is carried out with various pairs of center-symmetric features and local binary pattern features. Results show that the use of joint distributions of such a pair of features outperforms a single feature and the feature pair of LBPROT/SCOV provides the best results. In the second case, a comparative study is presented, the performance of their method is compared with the well-known circular symmetric autoregressive model (CSAR) (see Section 4.1) feature. It is shown that this approach can give better results than CSAR features. Although good results on given texture database have been reported in this paper, some problems have to be considered. Since most of these features are based on local window statistics, they cannot capture enough information of texture with strong global directionality. This may produce worse results when used for strongly ordered texture classification. The performance can be improved when the shape of the rotated local binary pattern is changed to another form (such as the circular shape) instead of the square shape.

4. Model based methods

Earlier attention in texture analysis is mainly focused on the first- and second-order statistics of textures. Later, many model-based methods (including Gaussian Markov random fields [18], Gibbs random fields [47], and Wold model [22,23]) are introduced to model texture. In these methods, a texture image is modeled as a probability model or as a linear combination of a set of basis functions. The coefficients of these models are used to characterize texture images. The key problem of these methods is how to estimate the coefficients of these models and how to choose the correct model suitable for the selected texture. The coefficients of these models are often transformed into different forms invariant to translation, rotation and scale.

4.1. SAR and RISAR models

Simultaneous autoregressive model (SAR) [17] has been used for texture classification, segmentation and synthesis by many authors.

Let $f(s)$ be the gray level value of pixel at site $s=(i,j)$ in an $N \times N$ textured image, $i,j=1,2,\ldots,N$. The SAR model is then defined as

$$f(s)=u+\sum_{r\in\omega}\theta(r)f(s+r)+\varepsilon(s), \tag{8}$$

where ω is the set of neighbors of the pixel at site s, $\varepsilon(s)$ is an independent Gaussian random variable with zero mean and variance σ^2, u is the bias independent of the mean gray value of the image, and $\theta(r)$ are the model parameters which can be used as texture features. The basic SAR model is dependent on the rotation as $\theta(r)$ also changes when the textured image is rotated.

From SAR, Kashyap and Khotanzad [17] have developed a circular simultaneous autoregressive (CSAR) model for invariant texture analysis based on circular neighborhoods. The method uses three statistical features, two of which are obtained from a new parametric model of the image. Two of the proposed features have physical interpretations in terms of the roughness and directionality of the texture. Two classes of texture images (including macrotextures and microtextures) are selected from the Brodatz album. Two tested image databases are constructed from the seven images of each class. Database 1 has 28 64×64 images per texture. Database 2 has 128 64×64 images from each texture. An average classification accuracy of 89% is reported on database 1, and 91% on database 2.

Although the CSAR can be used for invariant textures, however, the CSAR is defined on only one circle. This means that only the points on this circle around the central point are used to describe the CSAR. May be this is not enough to give an accurate description of the relationship between a pixel and its neighbors. Mao and Jain [107] develop the CSAR model into a multivariate invariant version named rotation invariant SAR (RISAR) model (see Ref. [107] for details). In their method, the neighborhood points of a pixel are defined on several circles around it. Thus, when the image is rotated around this pixel, the weighted gray values (defined as a function of the gray values of the points along each circle) at each circle remains approximately the same. This results in rotation invariance. The model parameters are extracted as the rotation invariant texture features.

The performance of the RISAR model is tested on seven textures selected from the Brodatz album. Each texture is rotated at seven different orientations. The highest correct classification rate obtained in this experiment is 87.9%, much higher than 60.1% acquired by the circular symmetric autoregressive (CSAR) model on the given textures.

Although the RISAR can give a better classification result than the CSAR model, the drawback of the utilization of the RISAR model still remains. It lies on two aspects. One is how to choose a proper neighborhood size in which pixels are regarded as independent, the other is how to select an appropriate window size in which the texture is regarded as being homogenous.

In order to overcome these difficulties, Mao and Jain [107] combine the multiresolution image representation and the RISAR model, and further develop the RISAR model into the multiresolution RISAR (MR-RISAR) model. The reason for this operation is that the fixed window size in different resolution images covers different region sizes in the original image. For the same given textures, the MR-RISAR model results in a classification rate of 100% at four resolutions.

4.2. Markov model

Cohen et al. [18] model texture as Gaussian Markov random fields and use the maximum likelihood to estimate coefficients and rotation angles. The problem of this method is that the likelihood function is highly nonlinear and local maxima may exist. In addition the algorithm must be realized by using an iterative method that is computationally intensive.

Chen and Kundu [19,76,77] address rotation invariance by using multichannel subband decomposition and hidden Markov model (HMM). They obtain the rotation invariant texture features using HMM in two stages. In the first stage, the quadrature mirror filter (QMF) bank is used to decompose the texture image into subbands. In the second stage, the sequence of subbands is modeled as HMM which is used to exploit the dependence of these subbands and is capable of capturing the trend of changes caused by the rotation. Two sets of statistic features are extracted from each subband. The first consists of the third- and fourth-order central moments normalized with respect to the second-order central moments. The second is composed of normalized entropy and energy. When texture samples are rotated, feature vectors derived from the original texture and its rotated version through the QMF bank are obviously different. Some problems have to be considered in their method. Variations of these feature vectors will increase as the number of texture samples of the same class grows. Although Chen and Kundu have reported that the HMM classifier can handle the variations, however when the number of texture classes increases, the performance of the HMM classifier may deteriorate due to the increasing variations. If these variations can be handled before using the HMM, the HMM classifier may produce better results.

In order to attack the problem, Wu and Wei [46] present a general method. In this method, a 1-D signal is obtained by resampling images along a spiral contour. The sampled (1-D) signal is then considered as the observation of some random process. After resampling a 1-D signal is passed through the QMF bank and features are extracted from each bank. The statistics of the resampled signal are extracted as the rotation invariant features. The gray-scale transform invariance can be achieved by a mean removal and a normalization technique.

Testing experiments are performed on 16 textures selected from the Brodatz album. Centers of tested images are randomly chosen from the 512×512 images and rotated to random angles. The highest correct classification rate of 95.14% is obtained, while the best classification rate of conventional second-order statistics is 88.39%.

Although good experimental results have been shown in Ref. [46], some problems still remain to be considered. In their method, texture images have to be spirally resampled, and the coordinates of the resampled points are quantized to the closest lattice points. There may exist some other interpolation methods (such as bilinear interpolation, cubic interpolation and spine interpolation, etc.) which can determine the resampled values more precisely. In the QMF subband decomposition, each subband is considered to have the same bandwidth. Maybe this is not the best choice. Since the signal energy is usually concentrated in the low frequency, subbands with unequal bandwidths may produce better results.

4.3. Wold-like model

Wold-like model is a more recent method used to model texture images. The model allows the texture to be decomposed into three mutually orthogonal components. The 2-D Wold theory applied to the textures is based on some approximations. It is assumed that texture images are homogeneous random fields. A texture image can be represented as the following decomposition [20]:

$$y(m,n) = w(m,n) + p(m,n) + g(m,n), \tag{9}$$

where $w(m,n)$ is the purely indeterministic component, $p(m,n)$ the half-plane deterministic component, and $g(m,n)$ the generalized evanescent component.

Francos, Narasimhan and Woods [20] present a maximum-likelihood solution to the joint parameter estimation problem of the three components from the single observed texture field.

The decomposition of the spatial field can also be achieved by the decomposition of their spectral density function (SDF) through a selected global threshold [20,21], but the global threshold method does not work for many natural textures in the Brodatz database. Liu and Picard [22,23] propose a new Wold based model to extract Wold texture features. These features which preserve the perceptual property of the Wold component are extracted without having to decompose each original texture.

Wu and Yoshida [109] describe a new method of using the Wold model for invariant texture analysis. In their method, texture images are decomposed into a deterministic field and an indeterminstic field. The SDF of the former is a sum of 1- or 2-D delta functions. The 2-D autocorrelation function (ACF) is used to model the indeterministic component. The parameters of ACF are extracted by using the least squares method. These parameters combined with the positions of the 2-D delta functions and the directions of the 1-D delta functions are used to produce rotation invariant texture features.

Eighteen Brodatz texture classes are selected and tested in the experiment. The experiment is performed in two cases. In the first case, texture images undergo only rotations (12 samples per class). The correct classification rate is 99.1% in the first case. In the second case, texture samples are both scaled and rotated (30 samples per class). And the correct classification rate of 91.3% is obtained.

Liu and Picard [22] have reported that the relationship of harmonic peaks (see Ref. [22] for details) on the SDF of the deterministic field may be unchanged under translation, rotation and scaling. Whether and how the property can be used for affine invariant texture analysis needs further study.

4.4. Multichannel Gabor filter

The 2-D multichannel Gabor filter is a windowed signal processing methods [29,34,35,71,75]. It can be used for texture analysis for several reasons: they have tunable orientation and radial frequency bandwidths, tunable center frequencies and optimally achieve joint resolution in spatial and frequency domain.

Bovic, Clark and Geisler [24,25] give a very detailed analysis of the Gabor function using localized spatial filters for texture feature extraction.

A typical 2D Garbor model used in the texture analysis is given as follows:

$$h(x,y) = g(x,y)\mathrm{e}^{-2\pi j(u_0 x + v_0 y)}, \tag{10}$$

where (u_0, v_0) is the center frequency and the function $g(x,y)$ the Gaussian function (here we assume that $g(x,y)$ is isotropic):

$$g(x,y) = \frac{1}{2\pi\sigma}\mathrm{e}^{-(1/2)[x^2+y^2/\sigma^2]}. \tag{11}$$

A set of Garbor filters and mental transforms are combined to obtain invariant texture features by Leung et al. [102] and Porter et al. [63]. Invariant moment features [102] are also studied in comparison with the Gabor features that are found to have better performance on invariant texture classification.

In the experiment reported in Ref. [102], 18 test sets consisting of 256 feature vectors are obtained from 6 different orientations (0, 30, 60, 90, 120, 150) and 3 different scaling factors $(1, 0.67, 0.5)$ to get an overall error rate of 10%, when log/polar Gabor filters and mental transforms are applied.

Haley and Manjunath [26] modify the Gabor model into the form of a polar 2-D Gabor wavelet. A texture here is modeled as a multivariate Gaussian distribution of macrofeatures. Microfeatures are spatially localized by multiresolution filters based on the polar form. Macrofeatures are derived from the statistics of the microfeatures. Both of them are invariant to rotations.

Thirteen textures from the Brodatz album and other sources are selected for testing the performance of this method. Each texture is rotated at $0^\circ, 30^\circ, 60^\circ, 90^\circ, 120^\circ$, 150°, and 200°. The MAP criterion is used to obtain a correct classification rate of 99.0%.

Tan and Fountain [27,28,48,49,51,65–70] also use Gabor filters to obtain the invariant texture features. The image is filtered into a set of cortical channel outputs. Each channel is modeled by a pair of real Gabor filters which are of opposite symmetry and are viewed as the function of two parameters: a radial frequency and orientation. For a fixed radial frequency, a rotation of the input image results in the translation of the average output by the same angle along the orientation axis. The magnitudes of the DFT of the sequence of the channel output energies are invariant to translations. Application of the DFT at each fixed frequency provides a set of invariant texture features.

Experiments in Ref. [51] on 300 texture images of 15 natural textures taken from the Brodatz album is performed to obtain an average correct classification rate of 89.3%. This result is achieved by using the first three features derived from the 16 channels at a single frequency.

Comparative studies performed by Randen et al. [32,64] and Chen et al. [33] indicate that the Gabor features in most of the cases outperform the other methods (ring/wedge filter, spatial filter, quadrature mirror filter, eigenfilter, wavelet transform) regarding the complexity and overall error rate mentioned in their literature. But the Gabor features suffer from a number of difficulties. A major difficulty of this method is how to determine the number of Gabor channels at the same radial frequency and the size of the Gabor filter window in the application. This selection can be guided by biological data from experiments with animal visual systems [24]. The optimal choice of the minimum set of the specific filters is studied by Jain and Karu [36] using a neural network.

4.5. Steerable pyramid

Greenspan et al. [37] present a novel rotation invariant texture recognition system using the steerable pyramid model. Texture features are extracted from the outputs of the oriented filters. A feature curve $f_c(\theta)$ is defined across the orientation space for an input texture. The rotation of the input image corresponds to the translation of $f_c(\theta)$ along the orientation axis. Since the DFT magnitude is invariant to translations, the invariant feature can be provided by the discrete Fourier transform of $f_c(\theta)$.

Thirty texture classes from the Brodatz album are tested to demonstrate the performance of the rotation invariant system. Three classification schemes (the nearest neighbor algorithm, the BP neural-network algorithm, a rule-based network classifier) are utilized. The results show that the K-nearest neighbor algorithm always gives the highest correct classification rate among the three classifiers.

4.6. Wavelet transform

Wavelet transform is a very powerful model for texture discrimination [60]. The wavelet transform decomposes a texture image into a set of frequency channels that have narrower bandwidths in the lower frequency regions. The transform is suitable for textures consisting primarily of smooth components, so that their information is concentrated in the low frequency region.

Chang and kuo [90] use the tree-structured wavelet transform for texture classification. Porter [61,62] develops the wavelet transform for invariant texture analysis based on the Daubechies four-tap wavelet filter coefficients. In his method, the texture is decomposed into 10 channels, which are obtained through the 3 level wavelet decomposition. At each level, the texture is decomposed into four channels, which can be represented by LL, LH, HL, and HH. The LH channel, for example, represents information of a low horizontal and high vertical frequency. Since the HH channel contains most of the image noise, it is discarded at each decomposition level. Thus only seven channels remain. The horizontal and vertical components at each frequency are combined to get the rotation invariants.

Experiments on 16 Brodatz textures at six orientations are performed to test the performance of this method. The average wavelet coefficient is extracted from each of the remaining four channels and used as invariant features for the classification. The drawback of this method is that the directional information is lost when the channels are combined. This causes the majority of the misclassifications of 4%.

5. Structural methods

In structural methods, texture is viewed as consisting of many textural elements (called texel) arranged according to some placement rules. From Fig. 3, a human observer can strongly perceive the structural properties of some textures. This leads to many structural texture

Fig. 3. Examples of structural textures (selected from the Brodatz album).

analysis methods. The structural properties of these elements have been successfully applied to characterize the textures by many authors. Commonly used element properties are average element intensity, area, perimeter, eccentricity, orientation, elongation, magnitude, compactness, euler number, moments, etc. [40,100]. A structural method suits better for a description of a macro-texture. Structural texture methods have been extended to invariant texture classification. A difficulty of these methods is how to extract texels of a texture.

5.1. Perimeter contribution and compactness

Perimeter and compactness of a primitive are used to characterize texture by Goyal et al. [38,39,57]. Perimeter is in theory invariant to shift and rotation. But in practice, it varies due to quantization error. The conventional approach of a pixel counting method causes large computation errors on the estimation of the perimeter. To overcome this problem, a new method of perimeter calculation is proposed by Goyal et al. [40] to improve the value of the perimeter. A percentage of change is defined to evaluate the performance of this method. A comparative result shows that the value of the perimeter calculated by this method not only varies less under rotation but also produces a more accurate estimation of the element compactness.

Roundness or compactness of a primitive is defined as follows:

$$r = \frac{P^2}{4\pi S}, \tag{12}$$

where S is the area of the primitive, and P the perimeter. The compactness properly is not only invariant to translation and rotation, but also invariant to scaling. The method of extraction of such primitives of a texture affects texture features.

5.2. Invariant histogram

Histogram is a very useful method for texture analysis. A texture can be characterized by the histogram of its texture elements.

Goyal et al. [41] have noted that the texel property may not change under translation and rotation. They use the property histogram to get the invariant texture features and propose a method to convert the traditional property histogram to an invariant histogram in which the number of the texels (frequency) is plotted on the Y-axis and the property value on the X-axis. For a particular texel value, the weighted frequency f_W is given as follows:

$$f_w = \sum_{i=1}^{k} a_i n_i, \tag{13}$$

where a_i is the area of texel of area-index i, and n_i the number of elements of index i. Although a_i and n_i will change when a texture is scaled, the product of a_i and n_i remains the same, which gives the invariant histogram of a texture.

Ninty-six textures generated by 6 classes of textures from the Brodatz database undergoing 4 rotations and 4 different scales are tested within the same textures, between different textures, and on texture recognition. An average of all invariant histograms is used as the class representative histogram (CRH) to represent a class of textures of different scales and rotations. An average classification rate of 95% is obtained by using the correlation between different CRHs.

5.3. Topological texture descriptors

Eichmann and Kasparis [44] discuss the extraction of invariant texture descriptors from line structures based on the Hough transform (HT). Line segments in the image correspond to the points in the HT plane. Since traditional HT is accomplished in the binary image, they use the Radon transform to implement HT for nonbinary textures. Image rotation is equivalent to the translation in the HT plane along the angle axis. Scaling in the texture is equivalent to the translation in the HT plane along the ρ-axis. Three types of texture primitives are extracted from the HT domain. The texture feature vector is composed of the following components: (a) the number of line orientations, (b) the relative orientation angles, and (c) the line spacing for each orientation. Since the HT plane can be very 'noisy', median filtering is employed to eliminate local noisy peaks in the HT.

Experiments are performed to show that different views of the same textures always yield the same feature vector. Apparently, this method is only suitable for the textures composed of lines.

5.4. Morphological decomposition

Lam and Lin [45] use iterative morphological decomposition (IMD) method for scale invariant texture classification. When applying the IMD method, a texture is decomposed into a scale-dependent set of component images. Some statistic features (mean, variance, normalized variance, and gradient) are obtained for each component texture.

A database consisting of eight textures from the Brodatz album is tested for the scaling invariant texture classification. Three structural elements (square, horizontal, and vertical) are employed to obtain the granulometries. The best correct classification rate of 97.5% is obtained by using the horizontal and vertical elements.

The Lam and Lin method also suffers from the same difficulty as other structural methods. Only three restricted structural texels are extracted in their method. Employing other types of structural elements (such as a circular component) combined with the three may produce better results.

6. Conclusions and possible future work

Although many papers have been published on texture analysis, the definition of texture is still an open issue [98]. The definition of texture will strongly affect the choice of texture analysis methods.

From the study of the existing literature on invariant texture analysis, the following observations may be drawn:

1. Some statistic texture features are invariant to translation and rotation, including the global mean, variance and central moments.
2. Fourier spectrum may be used to deal with translation invariance.
3. Polar coordinates can be employed on the frequency domain to obtain rotation invariant features including the Gabor model, ring and wedge filter, harmonic expansion, etc. Combined with the log-polar representation, scale invariant texture features can also be extracted.
4. Some properties of the texels are invariant to translation and rotation, including area, perimeter, and compactness of the texels. The property histogram can also be used to obtain scale invariance.

So far, almost all existing methods on invariant texture analysis are restricted to translation, rotation, and scale invariance. Seldom authors [43,83,111] can be found on the true affine invariant (i.e. including the skew factor) texture analysis. More attention should be paid on invariant texture analysis under the affine model and perspective transform [58] in the future work. At the same time, more efforts should also be devoted to 3-D texture analysis [82,85,93]. Many new algorithms of texture segmentation and new texture features have been proposed in the recent literature [84,99,100,103,104]. Whether and how these new algorithms and texture features can be developed for invariant texture analysis should also receive more attention in the future work. It is worth pointing out that a great deal of work [50,56,59,80,95,97,105,106,112] has been done on geometric invariants for object recognition from perspective images. Whether and how such geometric invariants can be used for perspective invariant texture analysis is an interesting, open question.

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