# On Capturing 3D Isotropic Surface Texture using Uncalibrated Photometric Stereo

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*Abstract* - **We propose an uncalibrated method for acquiring the normal and albedo fields of an isotropic 3D surface texture illuminated at a constant slant angle. The method is 'uncalibrated' in that the illumination vectors are not known *a priori*. We assume single point lighting of a rough Lambertian surface lying in the *x-y* plane. We use Hayakawa's uncalibrated photometric stereo algorithm to simultaneously estimate the scaled surface normals and the illumination vectors in an arbitrary co-ordinate system. The use of constant illumination slant intensity data means that the required orientation to a viewer co-ordinate system simply involves a z-axis rotation. Orientation in the x-y plane is determined by applying a frequency domain method for estimating illumination tilt angles. Preliminary results from simulations and real data are provided.**

## I. Introduction

Photometric stereo (PS) can be used to acquire the normal and albedo fields of surface texture from a minimum of three intensity images [1]. In order to solve the equations for these unknowns, however, the illumination vector corresponding to each image must be determined. The fact that *a priori* knowledge is a prerequisite for this method limits its application.

Hayakawa [2] proposed a new PS method which avoids measurement of the illumination and is essentially uncalibrated. However, the resulting surface normals and illumination vectors are in fact defined in an arbitrary co-ordinate system. Determining the orientation requires prior knowledge of either three surface or illuminant vectors. Hayakawa's algorithm has been researched further by several authors and employed in the area of face recognition [7, 8, 9, 10]. Belhumeur et al [8] noted that a generalised bas-relief ambiguity exists and proposed using the integrability constraint to solve it. In doing so they limit the method to smooth surface applications. As we intend to apply the method to globally flat but rough surface textures this assumption is not applicable in our work. They also recommend the use of a general calibration object but this is not suitable for our intended application. Drobohlav [11, 12] has also tackled the inherent ambiguity in uncalibrated PS. He proposes reducing the ambiguity by considering both Lambertian and specular terms but couples this with the integrability constraint.

This paper presents preliminary work on developing uncalibrated PS for application to surface textures. We use Hayakawa's algorithm to determine the surface gradients and albedo image without calibration. We assume that the illuminant slant is unknown but constant. Furthermore we assume that the surface texture is isotropic in nature such that the illuminant tilt angle can be estimated from the intensity image.

## II. Theory

### A Photometric Stereo

The PS algorithm is derived from Lambert's law. Thus the irradiance of a surface element with surface normal, **n**, illuminated by a source with vector, **l** is:

$$i(x,y) = \rho(\mathbf{l} \bullet \mathbf{n})$$

where $\rho$ = surface albedo

$\mathbf{l} = (l_x, l_y, l_z) = (\cos\tau\sin\sigma, \sin\tau\sin\sigma, \cos\sigma)$

$\mathbf{n}$ = unit surface normal

$\tau$ = tilt angle (latitude),

$\sigma$ = slant angle (longitude)

If three images $i_1$, $i_2$ & $i_3$ of a surface are captured under different illumination conditions which are *known*, it is straightforward to solve the 3 equations for the *scaled surface normals*, $\mathbf{t} = \rho\mathbf{n}$:

$$\mathbf{t} = \mathbf{L}^{-1}\mathbf{i}$$

*where:*

$$\mathbf{L} = \begin{bmatrix} l_{1,x} & l_{1,y} & l_{1,z} \\ l_{2,x} & l_{2,y} & l_{2,z} \\ l_{3,x} & l_{3,y} & l_{3,z} \end{bmatrix} \quad \textit{and} \quad \mathbf{i} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

In calibrated PS **L** and **i** are known.

### B Uncalibrated Photometric Stereo

For uncalibrated PS only **i** is known. We therefore face a bilinear problem in which both **L** and **t** must be simultaneously estimated. This is a generic machine vision problem in which *calibration times estimation equals data* or **C.E=D** [6]. One solution is $\mathbf{C} = \mathbf{E} = \sqrt{\mathbf{D}}$ but it is not unique: there is an ambiguity such that $(\sqrt{\mathbf{D}}.\mathbf{A})(\mathbf{A}^{-1}\sqrt{\mathbf{D}}) = \mathbf{D}$. Other prior information is required to resolve this ambiguity.

Hayakawa's algorithm [2] for uncalibrated PS tackles the problem in exactly this way. A matrix of known data is constructed by inserting each captured image of the surface texture as a column.

$$\mathbf{I} = \begin{bmatrix} i_{11} & \dots & i_{1f} \\ . & \dots & . \\ i_{p1} & \dots & i_{pf} \end{bmatrix} \quad \text{where} \quad \begin{array}{l} p = \text{no. of pixels} \\ f = \text{no. of frames} \end{array}$$

Singular value decomposition (SVD) is applied to the resulting intensity matrix **I** to provide an initial estimate for the surface and illumination matrices:

$$\mathbf{I} = \mathbf{SL} = \mathbf{U\Sigma V}$$

Pseudo surface and illumination matrices are formed by taking the three largest eigenvalues $\Sigma'$ and the corresponding columns and rows in **U** and **V** respectively, $\mathbf{U}'$ & $\mathbf{V}'$, as follows.

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$$\hat{\mathbf{S}} = \mathbf{U}'(\pm\mathbf{\Sigma}')^{1/2} \qquad \hat{\mathbf{L}} = (\pm\mathbf{\Sigma}')^{1/2}\mathbf{V}'$$

The ambiguity, **A**, is then resolved by assuming that there are six pixels with constant albedo or six frames in which the intensity of the light source is constant.

$$\mathbf{S} = \hat{\mathbf{S}}\mathbf{A} \qquad \mathbf{L} = \mathbf{A}^{-1}\hat{\mathbf{L}}$$

*C Absolute Orientation*

The resulting surface and illumination matrices, **S** & **L**, are in an arbitrary 3D co-ordinate system and require alignment. Horn [3] details a method to perform this. However, it requires three surface normals or illumination vectors in the viewer-oriented system to be known. The use of a calibration object would make this possible but is not suitable for all applications.

We assume that the illumination slant angle is constant. Although its value is unknown, the fact that it does not change affects the results of the decomposition. Whilst the variation in the intensity data cloud is accounted for by two eigenvectors, one eigenvector provides a measure of the distance of that cloud from the origin i.e. the data's average. The elements of the pseudo illumination matrix column corresponding to this 'average' eigenvector are equal in the case of constant illumination slant. Since the z-component of the actual illumination matrix is also constant this means that the required orientation simplifies to a z-axis rotation. It is noted that this rotation may be augmented by a permutation but that this is dependent on the relative position of the eigenvectors. In any case, an approximation for the first and second elements of three illumination vectors is then all that is required to find the overall rotation matrix. The tilt angle corresponding to each is actually sufficient for this purpose since the slant angle has effectively been estimated.

*D Illuminant Tilt Angle Estimation*

It is possible to estimate the illuminant tilt angle for an isotropic texture from its image. Knill [5] proposed a spatial domain estimator to do so but we use Chantler's [4] frequency domain method due to accuracy considerations. In the latter, the polar plot $P(\theta)$ is determined from the Fourier transform of the image. The polar plot for an isotropic surface texture is a cosine function whose maximum value corresponds to the illuminant tilt angle (See Fig.1).

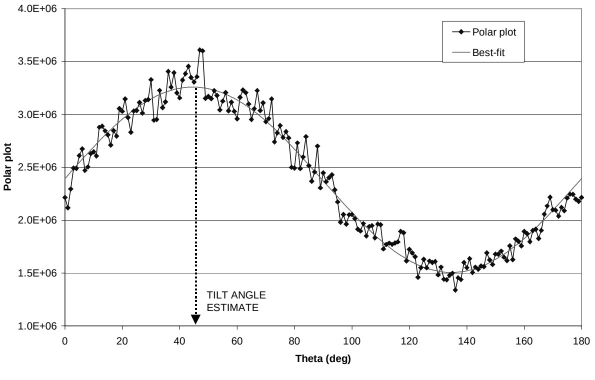


Fig. 1. Polar plot of synthetic isotropic surface illuminated from $\tau$ =45°, $\sigma$ =45°. Best-fit cosine shown - offset provides tilt estimate.

Fourier analysis is subsequently applied to the polar plot data in order to estimate the angle. It is given by the following formula:

$$\hat{\tau} = \left(90 - \tan^{-1}\left(\frac{a}{b}\right)\right)/2$$

where

$$a = \int_0^{\pi} P(\theta)\cos(2\theta)d\theta \qquad b = \int_0^{\pi} P(\theta)\sin(2\theta)d\theta$$

*E Summary*

A brief outline of the proposed overall method follows :

1. Capture intensity images of isotropic surface textures under constant illuminant slant angle conditions.
2. Process resulting images with Chantler's illuminant tilt estimator to predict tilt angle for each.
3. Process overall intensity matrix **I** with Hayakawa's uncalibrated photometric stereo algorithm.
4. Orient the resulting surface normal and illumination matrices to viewer-oriented 3D system using Horn's method with estimated tilt angles for three vectors.

## III. ASSESSMENT

*A Synthetic images*

We initially tested the proposed method with synthetic images to determine how well it worked under controlled conditions. We used a fractal model since it produces isotropic surfaces which look natural. The images were generated by rendering the resulting height map and used as input for our algorithm. The output (the surface normal fields) was compared against surface normal estimates obtained from the fractal height map. Accuracy was expressed as a signal to noise ratio.

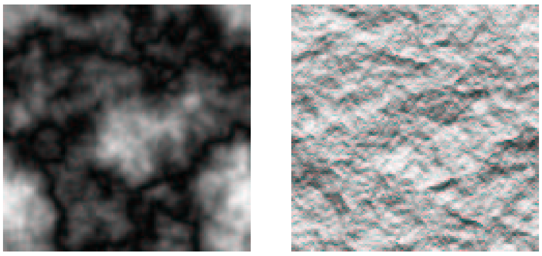

Fig. 2. Fractal height map ( $\beta$ =3.5), left, and an example of an image generated by relighting it, right.

Each basic data set consisted of 36 images. For a given illuminant slant angle, images were generated every 10° with regard to tilt angle. Further data sets were produced by adding increasing levels of white noise to simulate real conditions. Experiments were performed over a range of input signal to noise values with different illuminant slants and increasing height variation. The results are presented below.

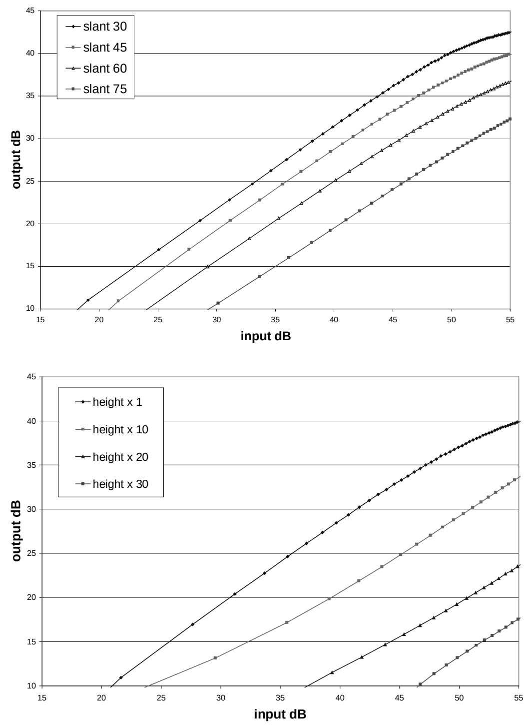


Fig. 3. Output signal to noise ratios against input for the synthetic fractal surface (a) slant variation, top and (b) height variation, bottom.

It is noted that the signal to noise ratio decreases with increasing slant angle; it also decreases with increasing height variation. This indicates that we should avoid extremely rough surface textures and furthermore avoid raking illumination conditions. Increasing the degree of input noise results in an increasingly noisy output.

### *B Real data*

In order to assess the performance of the proposed method with real data, we captured images of an isotropic texture under various illumination conditions. A fractured plaster surface was used. The texture was lit by a light source at a distance of approximately 1m and images of it were captured with a digital camera. It is noted that both the camera and the texture were fixed in position and orientation. The light source was moved and/or pivoted to provide illuminant orientations corresponding to 36 different tilt angles for three different slant angles. This resulted in three data sets, each of which was processed both by the calibrated PS algorithm as well as the uncalibrated method to allow comparison. The resulting surface gradients and albedo images were then used to generate relit images corresponding to the entire range of illumination conditions used during the image capture session. This meant that it was then possible to compute an average image signal to noise value for each method at each slant angle.

The relit images for both methods were found to be visually similar to the originals. However, the variance in the relit images corresponding to the completely uncalibrated method was significantly higher than that for both the original images and those generated from the calibrated results. A comparison between the surface gradients predicted by both methods revealed the cause to be a constant scaling factor; this linear relationship is clearly demonstrated in Figure 4. We believe that this effect can be attributed to the bas-relief ambiguity [7]. In this case a z-axis rotation alone is not sufficient to completely orient the data. In order to resolve this ambiguity, the scaling factor must be determined.

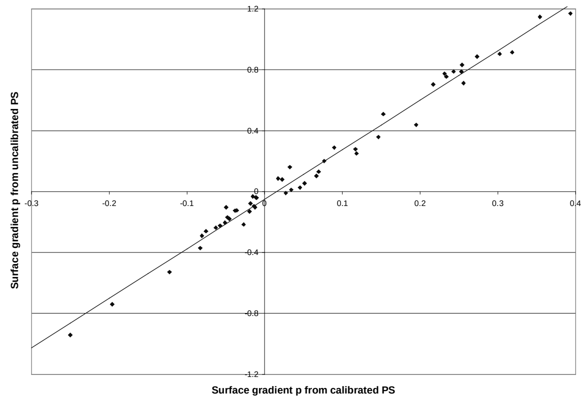


Fig. 4. Plot of surface gradient values generated by uncalibrated PS against corresponding values generated by calibrated PS for a real textured surface (best fit line also shown).

Since we are dealing with images of an isotropic texture illuminated with constant slant angle, the mean and variance will be constant for every image. This observation allows the images generated by relighting the uncalibrated data to be scaled and thus compared to the originals. Average signal to noise ratio values are given in the bar chart in Figure 5. A visual comparison of relit images is given in Figure 6. The uncalibrated results compare favourably with the calibrated results both visually and in terms of the signal to noise ratio.

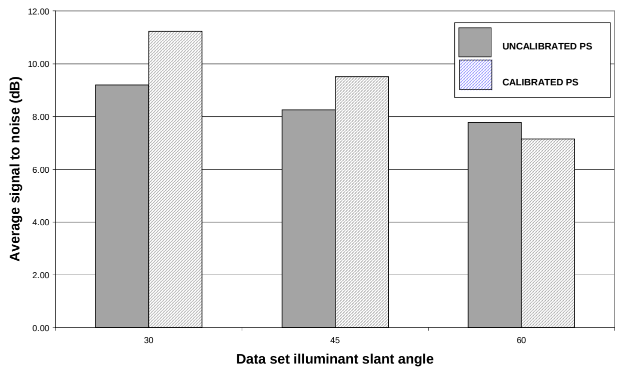


Fig. 5. Plot of surface gradient values generated by uncalibrated PS against corresponding values generated by calibrated PS for a real textured surface.

## IV. CONCLUSIONS

In this paper we have presented research the ultimate aim of which is to develop a completely uncalibrated photometric stereo algorithm for the acquisition of surface texture for use in relighting applications.

We have chosen to avoid the use of a calibration object and instead constrain the input data. Our current approach requires that the slant angle of all illumination vectors is a constant but unknown value. We also assume that the surface is isotropic and aligned with the x-y plane. As a result the orientation from arbitrary to viewer oriented co-ordinate system involves a rotation about the z-axis which is estimated using an existing frequency domain method.

Whilst limited in application, the resulting algorithm has been shown to work well in the absence of noise with isotropic images of synthetic textures. It also provides excellent 're-lights' of a real texture.

We intend to assess the performance of the method on a wide variety of surface textures and to extend the approach to deal with directional textures.

## REFERENCES


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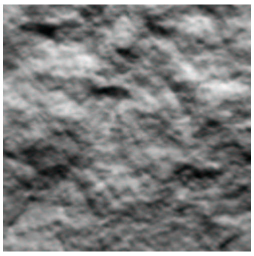
Fig. 6. Comparison of an original image with relit images which have been generated by using the surface gradient and albedo images determined by both the uncalibrated and the calibrated photometric stereo methods from data sets of various illuminant slant angles.

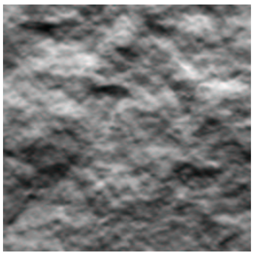
**A** Original image
$\tau = 90°$, $\sigma = 45°$

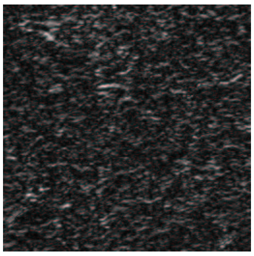


**I.** *UNCALIBRATED PS*

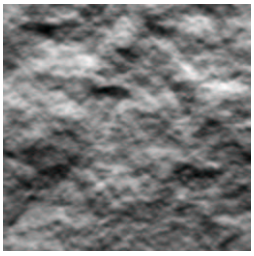
**B** Relit images **C** Error magnitude images

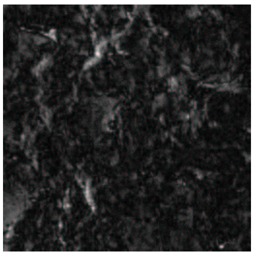
1. Original data set with illuminant slant angle of 30°



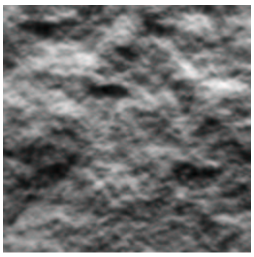


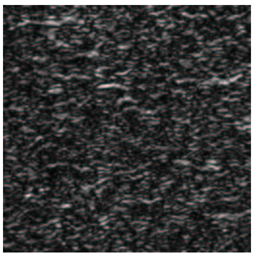
2. Original data set with illuminant slant angle of 45°





3. Original data set with illuminant slant angle of 60°

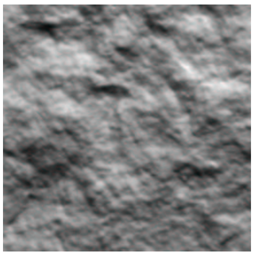


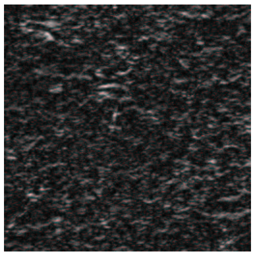


**II.** *CALIBRATED PS*

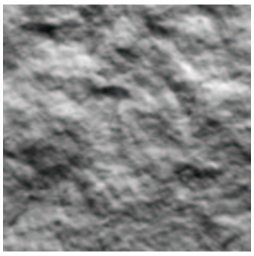
**B** Relit images **C** Error magnitude images

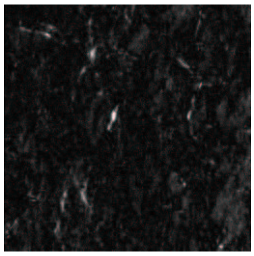
1. Original data set with illuminant slant angle of 30°





2. Original data set with illuminant slant angle of 45°





Original data set with illuminant slant angle of 60°